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Price index dispersion and utilitarian social evaluation

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Abstract

The living standard indicator in utilitarian social evaluation functions (USEF) is the ratio of a nominal living standard and a price index. We show that under weak association of price indices and nominal living standards and usual concavity conditions on utility functions, utilitarian social welfare increases with price index dispersion when the aggregate price level is superior to the arithmetic mean of price indices, and diminishes when it is inferior to the harmonic mean.

Keywords: Personal income distribution; Economic welfare; Price dispersion

JEL classification: D31; D6

1. Introduction

In social evaluation functions, the decomposition of living standard indicators is usually done by using additive specifications, to distinguish several sources of income or risk.[1] However, living standard indicators can be better seen as a nonlinear combination of components: on one hand, income or consumption data, and on the other, prices, household characteristics and environment.

What is the impact of price dispersion on social welfare? To deal with this question, we study the consequences of the ratio functional form for living standard variables in utilitarian social evaluation

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[1] E.g. Kihlstrom et al. (1981).

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function (USEF), which are the only social evaluation functions satisfying some attractive axioms.[2] Moreover, our results are valid under 'generalized utilitarianism'.[3]

If the price deflation is inaccurate, then apparent welfare differences between households may come from price differences.[4] Therefore, welfare policies may be seriously misled by non-deflated indicators. The correlations between components of the living standard and their dispersions play complementary roles. We focus in this paper on their dispersion and on cases where the numerator and denominator of the living standard variable are weakly statistically linked.

The influence of spatial price deflation on social welfare has not attracted much attention in the theoretical literature.[5]Price indices are extensively studied in the theoretical literature.[6] However, we do not directly deal in this paper with substitution effects in price indices.

The results that we present are useful firstly because they help understand the impact of the distribution of price indices on social welfare. This may be useful to study how social welfare is related to price variability, how to model real living standards or social welfare analysis, and how to deal with missing information in prices. Secondly, the results reveal cases in which the price index dispersion is socially advantageous or noxious. Thirdly, they exhibit the special roles of harmonic and arithmetic means of price indices in welfare analysis.

2. The result

In welfare analysis, accounting for price differences across households implies that the living standard indicator or household i is (x_i/I_i), where I_i is the price index and x_i is the nominal living standard associated with household i. I_i is assumed to be strictly positive and x_i to be a real number.

The USEF W can be defined as:

$$W = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} u\left(\frac{x}{I}\right) \mathrm{d}F_2(I|x)\mathrm{d}F_1(x), \tag{2.1}$$

where F_1 is the marginal c.d.f. of the nominal living standard and F_2 is the c.d.f. of the price index conditional on the nominal living standard, u is the social utility function that is assumed measurable and increasing ($u(y)$ represents the welfare of an individual with real living standard y) and twice differentiable to facilitate calculations. W is defined over the set of probability measures on $R \times R_{++}$ where R_{++} is the set of strictly positive real numbers. In all this paper we assume that all the considered integrals are finite, which is satisfied with actual data and usual functional forms or u.

[2] See for example Chakravarty (1990). Thus, any welfarist ex-ante social evaluation functional satisfying anonymity and the weak Pareto principle is Utilitarian.

[3] In this setting, the function u (see below) may be any increasing function of the utility (Maskin, 1978).

[4] In this paper, we examine the price dispersion such that it appears through the dispersion of price indices. Indeed, price indices are sufficient statistics for the calculation of real living standards when nominal living standards are known. Changes in price dispersion cross products are not treated, even if they contribute to changes in dispersion of price indices. Moreover, we do not deal in this paper with the already studied effect on individual welfare of the instability in individual prices (Turnovsky et al., 1980; Ebert, 1994). We emphasize that, although these papers deal with a similar topic, their results rest on different mathematical bases and are not directly related to ours.

[5] Roberts (1980), Slivinski (1983) and Blackorby et al. (1999) examine when welfare prescriptions can be independent from the price configuration in the economy. They find it impossible, except for unsatisfactory welfare indicators.

[6] E.g. Baye (1985), Diewert (1990).

We next compare the USEF without price deflation ($I=1$ for all households), with the USEF accounting for the price index distribution (using deflated living standard indicators, x/I).[7]

Definition 2.1. The variation in the USEF caused by the price deflation is

$$\Delta W = \int_{-\infty}^{+\infty} \int_{0}^{+\infty} u\left(\frac{x}{I}\right) \mathrm{d}F_2(I|x)\mathrm{d}F_1(x) - \int_{-\infty}^{z} u(x)\mathrm{d}F_1(x). \tag{2.2}$$

No additional normative condition, other than when income is considered alone, is required to obtain our results. In particular, under a condition of impartiality or anonymity, the Pigou–Dalton transfer axiom is equivalent to the concavity of the social utility function.[8] To derive results relatively to an aggregate price level defined as the arithmetic mean, we need to consider the function $K_x(I) \equiv u(x/I)$. K_x is convex if and only if the relative risk aversion coefficient (RRAC) associated with $u(y)$ (equal to $-y \cdot u''/u'$) is inferior or equal to 2. K_x is concave if and only if the RRAC≥ 2.

When assumed in the case where u is understood as individual's von Neumann–Mongenstern utility, the normative justification of the convexity of K_x is a weak assumption. Indeed, it is satisfied, for example, or $u(x)=x^{\alpha}$, $\alpha>0$ and for $u(x)=\ln x$. It also corresponds to empirical estimates. Using US data, Gourinchas and Parker (2002) estimate the RRAC to be between 0.4 and 1.5. Because the ratio function ($1/I$) is very convex, only very substantial concavity u can generate K_x non-convex in I. We now concentrate on the case of 'weak statistical association' of nominal living standards and price indices, defined as follows.[9]

Condition C1. x and I are said to be weakly statistically associated at the numerator for the USEF if

$$\int_{-\infty}^{+\infty} u\left[x\int_{0}^{+\infty} \frac{1}{I}\mathrm{d}F_2(I|x)\right]\mathrm{d}F_1(x) = \int_{-\infty}^{+\infty} u\left[x\int_{0}^{+\infty} \frac{1}{I}\mathrm{d}F_2(I)\right]\mathrm{d}F_1(x)$$

$$= \int_{-\infty}^{+\infty} u(x/H)\mathrm{d}F_1(x), \text{ where } H \text{ is the harmonic mean of price indices.}$$

Condition C2. x and I are said to be weakly statistically associated at the denominator for the USEF if

$$\int_{-\infty}^{+\infty} u\left[\frac{x}{\int_{0}^{+\infty} I\mathrm{d}F_2(I|x)}\right]\mathrm{d}F_1(x) = \int_{-\infty}^{+\infty} u\left[\frac{x}{\int_{0}^{+\infty} I\mathrm{d}F_2(I)}\right]\mathrm{d}F_1(x) = \int_{-\infty}^{+\infty} u\left(x/\bar{I}\right)\mathrm{d}F_1(x),$$

where $\bar{I}$ is the arithmetic mean of price indices.

The conditions, without apparent normative meaning, state that the USEFs can be defined by using aggregate price indices instead of price indices specific to each living standard level.[10] The

[7] Eq. (2.2) may also describe the situation where x is a living standard indicator for which crude or non up-to-date price index has been used, while x/I corresponds to more accurate deflation. Note that it is not true that the price index can be chosen or renormalized arbitrarily since the real living standards and the corresponding price index must have normative sense.

[8] Atkinson (1970), Rothschild and Stiglitz (1973).

[9] Levy and Paroush (1974), Huang et al. (1978) are examples of uses of still stronger independence assumptions in welfare analysis.

[10] Note that our problem differs from multidimensional welfare analysis in which the utility function would admit (x, I) as argument (Atkinson and Bourguignon, 1982). First, there is no direct ethical property of u attached to variable I. A multivariate approach based on a joint generalized concavity in (x, I) would imply normative conditions hard to justify directly (e.g. given sign for u_{xI}). This would have little sense in our case. Second, multivariate approaches do not allow us to void hypotheses C1 and C2 necessary to make explicit the effects of price dispersion on welfare in terms of aggregate price indices (H or $\bar{I}$) independent from the nominal living standard distribution.

following sufficient conditions respectively or C1 and C2 are weaker than the independence and do not depend on u.

Condition C3. $\int_0^{+\infty} \frac{1}{I}\ \mathrm{d}F_2(I|x) = \int_0^{+\infty} \frac{1}{I}\ \mathrm{d}F_2(x)$ for all x almost surely, i.e. $E\ (1/I|x)=E(1/I)$ for all x almost surely.

Condition C4. $\int_0^{+\infty} I\ \mathrm{d}F_2(I|x) = \int_0^{+\infty} I\ \mathrm{d}F_2(x)$ for all x almost surely, i.e. $E(I|x)=E(I)$ for all x almost surely.

C3 implies that the coefficient of x in the regression of $1/I$ on x is non-significant. C4 implies that the coefficient of x in the regression of I on x is non-significant.

Although independence restrictions for several sources of risk are not rare in theoretical welfare or risk analyses (Kihlstrom et al., 1981), some justification of C3 and C4 is useful beyond the insight obtained by looking at a polar case. There exists empirical and theoretical support or C3 and C4. First, x and I may be weakly associated because of strong market imperfections disconnecting incomes and prices. Second, we find in Muller (2002) that the independence of I and x cannot be rejected for rural Rwanda. This is not an isolated result. In Russia during the latter part of the 1980s, the changes in price levels and in nominal wages have been found to be unrelated (Koen and Phillips, 1993). Even when the link between x and I is statistically significant, we do not expect it to be strong. To this extent, the case of weak association provides useful approximative insight. We obtain:

Proposition 2.2. *(a) Under* C1 *if u is concave,*

$$W \leq \int_{-\infty}^{+\infty} u(x/H)\mathrm{d}F_1(x) \leq u(\bar{x}/H). \tag{2.3}$$

(b) Under C2 *if the RRAC≤2 over the domain of the real living standards,*

$$W \geq \int_{-\infty}^{+\infty} u\left(x/\bar{I}\right)\mathrm{d}F_1(x). \tag{2.4}$$

(c) Under C2 *if the RRAC≥2 over the domain of the real living standards,*

$$W \leq \int_{-\infty}^{+\infty} u\left(x/\bar{I}\right)\mathrm{d}F_1(x) \leq u\left(\bar{x}/\bar{I}\right) \text{ [if moreover } u \text{ is concave].} \tag{2.5}$$

Proof.

(a) $W \leq \int_{-\infty}^{+\infty} u[\int_0^{+\infty} \frac{x}{I} \mathrm{d}F_2(I|x)]\mathrm{d}F_1(x)$ (Jensen's inequality applied to u) $= \int_{-\infty}^{+\infty} u[x \int_0^{+\infty} \frac{1}{I} \mathrm{d}F_2(I)]\mathrm{d}F_1(x) = \int_{-\infty}^{+\infty} u(x/H)\mathrm{d}F_1(x)$ (by C1 and definition of H)$\leq u(\bar{x}/H)$ (Jensen's inequality applied to u).
(b) $W \geq \int_{-\infty}^{+\infty} u[\frac{x}{\int_0^{+\infty} I\ \mathrm{d}F_2(I|x)}]\mathrm{d}F_1\ (x)$ (Jensen's inequality applied to K_x) $= \int_{-\infty}^{+\infty} u(x/\bar{I})\mathrm{d}F_1(x)$ (by C2 and definition of $\bar{I}$).
(c) $W \leq \int_{-\infty}^{+\infty} u[\frac{x}{\int_0^{+\infty} I\ \mathrm{d}F_2(I|x)}]\mathrm{d}F_1(x)$ (Jensen's inequality applied to K_x) $= \int_{-\infty}^{+\infty} u(x/\bar{I})\mathrm{d}F_1(x)$ (by C2 and the definition of $\bar{I}$)$\leq u\ (\bar{x}/\bar{I})$ (Jensen's inequality applied to u if moreover it is concave). □

The deflated USEF can therefore be majorized and minorized by USEFs calculated without price dispersion as soon as the aggregate level of prices is adequately defined. Since $\bar{I} \geq H$, Eqs. (2.3) and (2.4) can be combined to provide, under the assumption of RRAC < 2, an upper bound and a lower bound for the USEF that are based on the sole observation of $\bar{I}$ and H. This is useful when the distribution of prices is unknown, while the values of $\bar{I}$ and H are available or can be inferred.[11] Since observed $\bar{I}$ and H are generally close, the domain of aggregate price level for which there remains ambiguous results is likely to be narrow. In the case of Rwanda and Laspeyres indices in four successive quarters of 1982–1983,we found $\bar{I}$ equal to 1.028, 1.058,1.051 and 1.075, and respectively H equal to 1.015, 1.043, 1.034, 1.065 (Muller, 2002). Consider the special case when the distribution of price indices is lognormal. Then, if ln $\bar{I}$~$N(\mu, \sigma^2)$, we have $H = e^{\mu - \sigma^2}/2$ and $\bar{I} = e^{\mu + \sigma^2}/2$. Besides, the Theil index of inequality of the price index in that case is $T = \sigma^2/2 = (\ln \bar{I} - \ln H)/2$.

Proposition 2.2 can be used to express the effect of price dispersion at a constant aggregate level of prices. By stochastic dominance, under C1 and u concave, the effect of price dispersion at any constant aggregate price level equal or below H is negative. If on the contrary the price dispersion is defined in reference to a constant aggregate price level equal or greater than $\bar{I}$, then under C2 and RRAC ≤ 2, the effect of the price dispersion on welfare is positive, even when u is not concave. Finally, under C2 and a constant aggregate price level greater than $\bar{I}$, but with RRAC ≥ 2, the effect of the price dispersion becomes negative.

When $\bar{I}$ is fixed, then with RRAC < 2, the dispersion of price indices raises the level of the USEF. This result stems from the asymmetric shape of the inverse function, implying that the impact of a larger spread of price indices is stronger for a fall in prices than for an augmentation.

Consider two people who, in situation A, have living standards respectively of levels 1 and 1 (e.g. their fixed wages) and facing price indices equal respectively to 2 and 2. Suppose that after further observation we discover that prices must be corrected so that in situation B the people now ace price indices respectively equal to: $2 - 1 = 1$ and $2 + 1 = 3$. Clearly, $\bar{I}$ has not changed. The real living standards in situation B are respectively equal to 1 and 1/3. Although it depends on the risk aversion that one considers, many observers would agree that the first person situation has improved much more than the second person situation has deteriorated. For example, if $u(y) = \ln y$, $W(A) \simeq -1.38 < W(B) \simeq -1.09$; if $u(y) = \sqrt{y}$, $W(A) \simeq 1.41 < W(B) \simeq 1.57$.

With precautions, the theoretical results of this paper can be extended to equivalence scales, other functional forms and statistical conditions or the disaggregation of the living standard variable, several factors, inequality and risk analyses.

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[11] Unrelated approaches to derive bounds for poverty are in Bradbury (1997) and for social welfare in Fleurbaey et al. (2003). In these papers, the authors start from bounded intervals of equivalence scales and calculate the corresponding bounds of social welfare.

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