Available online at www.sciencedirect.com
SciVerse ScienceDirect
Journal of Economic Theory 147 (2012) 1427–1449
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Multidimensional inequality comparisons: A compensation perspective ☆

Christophe Muller [a], Alain Trannoy [b,*]

[a] *DEFI, AMSE, University of Aix-Marseille, France*
[b] *AMSE–EHESS, Marseille, France*

Received 25 April 2010; final version received 28 February 2011; accepted 29 August 2011

Available online 22 September 2011

Abstract

We provide a unified treatment of the two approaches pioneered by Atkinson and Bourguignon (1982, 1987) [3,4] by resorting to compensation principles in the bivariate case. We treat the attributes of individual utility asymmetrically by assuming that one attribute can be used to compensate another. Our main result consists of two sufficient second-order stochastic dominance conditions. In the case where the compensated variable has a discrete distribution, the distribution of the compensating variable must satisfy a condition which degenerates to the Sequential Generalized Lorenz test for identical marginal distributions of the compensated variable. Furthermore, the distributions of the compensated variable must satisfy the Generalized Lorenz test.

JEL classification: D3; D63; I31

Keywords: Multidimensional welfare; Compensation; Dominance; Lorenz criterion

☆ We are grateful to the British Academy and to the Centre National de la Recherche Scientifique for grant No. APN30649 that made this project possible. The first author acknowledges ESRC grant No. R000230326. We thank Steve Bazen, Pierre-Henri Bono, Jean-Luc Prigent, John Weymark and participants of seminars at GREMAQ in Toulouse, THEMA in Cergy-Pontoise, WIDER in Helsinki, Alicante and Barcelona, as well as the editors and the two referees for their comments and suggestions that have helped us to greatly improve the exposition and the content of the paper. The usual disclaimers apply.

* Corresponding author.
E-mail addresses: christophe.muller@univmed.fr (C. Muller), alain.trannoy@univmed.fr (A. Trannoy).

0022-0531/$ – see front matter
doi:10.1016/j.jet.2011.09.002

1. Introduction

The seminal contributions of Kolm [25] and Atkinson [2] have given rise to a large literature devoted to the quest for stochastic dominance theorems applied to welfare economics. While a well-organized corpus of stochastic dominance theorems is available in the unidimensional case, deriving social dominance conditions for multidimensional settings remains a major challenge in modern welfare analysis. Social scientists and economists (such as in Sen [33]) argue that income is not adequate as a measure of individual well-being and should instead be supplemented with other well-being attributes such as health and education. Income varies over time and comparing intertemporal income streams is another example of a multidimensional context. The statistical units used in most survey sampling frames are households rather than individuals. This implies that differences in family composition should be taken into account when dealing with welfare comparisons. Our first contribution is to offer an integrated framework for the multidimensional stochastic dominance literature applied to welfare analysis, which has developed in two ways: the multidimensional dominance approach and the needs approach.

In the first of these approaches, which can be traced back to Kolm [26], all attributes are treated symmetrically. In particular, for bivariate distributions, Atkinson and Bourguignon [3], henceforth AB1, proposed dominance criteria for classes of utility functions defined by the signs of their partial derivatives up to the fourth order.[1] Nevertheless, it seems fair to say that no simple criterion of multidimensional dominance has yet achieved general support among applied economists and even among theorists. This lack of success partly stems from the limited appeal of certain conditions imposed on utility functions. Up to now, there are no broadly accepted normative conditions for multidimensional stochastic dominance analysis, in contrast with normative conditions justified by transfer axioms for unidimensional stochastic dominance.

The landmark article by Atkinson and Bourguignon [4], henceforth AB2, is at the origin of the needs approach. In this case, the two attributes no longer have symmetric roles and the emphasis is on measuring income inequality while accounting for the heterogeneity of household needs, such as that stemming from differences in family size. One attribute (e.g., family size) is used to categorize the population into homogeneous groups, while well-being is derived from a second attribute (income). AB2 provided a simple and elegant procedure for performing welfare comparisons in this context: the *Sequential Generalized Lorenz* (SGL) quasi-ordering, which extends the *Generalized Lorenz* (GL) quasi-ordering (Shorrocks [34]) to situations where the population is partitioned into subgroups on the basis of needs.[2]

In the original paper AB2, the marginal distribution of needs is assumed to be the same in the situations being compared. Subsequently, Jenkins and Lambert [24] and Chambaz and Maurin [10] showed how the SGL test can be extended to the case where distributions of needs differ, albeit at the cost of an additional restriction on utility levels. Moyes [28] and Bazen and Moyes [7] have modified this additional restriction in order to allow the marginal distribution of needs to play a role in the comparison. Doing this makes the two approaches less distinct and questions the relevance of separating them.

Despite a kind of division of labor between the two approaches, it would be useful if they could be formulated using a common framework. As mentioned above, the first aim of this paper

[1] The interpretation of the signs in these characterizations is discussed in Moyes [28] for social welfare analysis and in Eeckhoudt, Rey and Schlesinger [16] for decisions under multidimensional risk by referring to risk aversion, prudence and temperance.

[2] For developments of the needs approach, see Fleurbaey, Hagneré and Trannoy [20] and Shorrocks [35].

is to provide such an integration for bivariate distributions. To do this, we need to derive results both for the continuous case and for the case of a discrete distribution of needs. This is because many welfare attributes we can think of have continuous distributions, while typically needs distributions are discrete. Furthermore, we do not assume that the marginal distributions for the compensated variable are fixed and we assume that this variable has cardinal significance.

In typical welfare comparisons with the first approach, marginal utilities are generally assumed to be: (1) identical across agents with respect to each attribute and (2) non-negative and non-increasing. However, these assumptions do not generally suffice to generate criteria with high discriminatory power. As a response to this issue, we introduce new assumptions based on compensation principles.

Recent contributions in distributive justice, surveyed in Roemer [31] and Fleurbaey [19], put forward ethical grounds for compensating for lower levels of certain attributes.[3] A frequent claim in the literature is that welfare differences are acceptable if they are due to attributes for which agents can be held responsible. In contrast, individuals should be compensated for a deficiency in other attributes. The debate about how to specify these two sets of attributes is far from closed (Dworkin [14]). Dworkin proposed to include preferences in the former category and resources (including inner resources like innate ability) in the latter. Atkinson and Bourguignon [5, p. 46], who allude to the possibility of compensation, seem to endorse Dworkin's position: "Differences in innate abilities, needs or handicaps would seem to require some kind of compensation, but not differences in effort, resulting from differences in tastes or preferences."

Our second contribution lies in the application of compensation principles through the assumption that at least one attribute can be used to make direct compensating transfers between individuals. For example, in the case where income and health are the only two attributes, income is the compensating variable and cash transfers could be applied in order to compensate one individual for a bad health status. Clearly, current income may have the same compensating role in many situations when other attributes are seen as being the compensated variables (e.g., past income, previous generation's income, health, education, family size and so on). This perspective justifies treating attributes asymmetrically, thus allowing us to extend the set of the normative conditions that are imposed and thereby potentially improving the discriminatory power of the dominance criteria that are used.

We introduce two compensation assumptions. First, compensation is good for social welfare. Second, compensation should be focused on those who are handicapped or needy and are in the lower tail of the income distribution.

We derive two sufficient conditions for a distribution of attributes to dominate another under these assumptions. First, for discrete realizations of the compensated attribute, the distributions of the compensated variable have to satisfy an absolute poverty gap test. The income poverty gap is cumulated for all individuals with income lower than some poverty line and compensating variable lower than a given threshold. The cumulated poverty gap thus obtained must be lower for the dominating distribution than for the dominated one, and this must be satisfied for any level of the poverty line and of the threshold. Second, the distribution of the compensated variable has to satisfy the GL test. Thus, to achieve dominance in the income-health example, that is, in order to improve social welfare by moving from joint-distribution A to joint-distribution B, it is sufficient that: (1) the income distribution from A dominates the income distribution from B in terms of

[3] Schokkaert and Devooght [32] review empirical results suggesting that the notion of compensation for 'uncontrollable factors' find some echo from many respondents in several countries.

the poverty gap test and (2) the health distribution from A GL-dominates the health distribution from B.

This result provides a simple (sufficient) test for a social welfare improvement in multidimensional settings. It is also attractive on two additional grounds. First, it is in line with the typical criterion obtained in needs analysis. Second, it corresponds to dominance for utility functions that have intuitive ethical meaning.

Our contribution may also be placed in the context of the recent literature on multidimensional stochastic orders. Starting with our class and dropping our second compensation assumption, we obtain a less discriminatory class, which corresponds to the 'increasing directionally concave' functions of degree 2 (denoted by IDIC_2), studied by Denuit and Mesfioui [13].[4] By dropping instead the assumption that the marginal utility of the compensating variable is decreasing, we obtain the class considered by Bazen and Moyes [7]. Our class appears to be the next in the sequence of more discriminatory classes for which the conditions of stochastic dominance were unknown. The fact that no complete results are available for IDIC_2 in the highly technical literature on stochastic orders illustrates the difficulty of deriving necessary and sufficient stochastic dominance characterization in the bivariate case. This justifies looking for sufficient partial derivative conditions when necessary and sufficient conditions cannot be readily found.

To sum up, we provide two contributions to the existing literature on welfare and inequality measurement: First, for bivariate distributions, we integrate the multidimensional dominance approach with the needs approach by dropping the assumptions of discrete values and fixed marginal distributions for the compensated variable, and by assuming that it has a cardinal meaning. Second, we explore the implications of adopting a compensation perspective in defining admissible signs for the partial derivatives of the utility functions. By considering that transfers in a given attribute can compensate for deficiency in another, we move away from requiring the 'symmetric' treatment of attributes, and in particular from assuming symmetric signs for partial derivatives.

The paper is organized as follows. The next section presents the setting and compares our class with those studied in the literature. Section 3 derives our sufficient conditions. Section 4 presents conditions in terms of inverse stochastic dominance. We discuss the introduction of additional transfer sensitivity conditions in Section 5 before presenting our conclusions in Section 6. The proofs of the propositions appear in Appendix A.

2. The setting

We consider the bivariate distribution of a variable $X = (X_1, X_2)$, where subscript 1 is used for the compensating attribute and 2 for the compensated attribute. We assume that the support of X is the rectangle $[0, a_1] \times [0, a_2] = A_1 \times A_2$, where a_1 and a_2 are in $\mathbb{R}_+$. This assumption encompasses most variables used in empirical work. Note that it implies that each variable has a cardinal meaning, which may be not satisfied in some contexts. $F(x_1, x_2)$ denotes the corresponding joint cumulative distribution function, $F_1(x_1)$ and $F_2(x_2)$ the respective marginal cdfs of X_1 and X_2 and F_1^2 is the cumulative distribution function of X_1 conditional on X_2. Since in practice F_1 is generally the distribution of incomes, typically considered as continuously distributed in economics, we assume that F_1 and any conditional distribution F_1^2 of income may be any non-negative, strictly increasing and continuous functions with range $[0, 1]$. F_2 can be any

[4] See also Denuit, Lefèvre and Mesfioui [12] and Denuit, Eeckhoudt, Tsetlin and Walker [11].

non-negative, non-decreasing and right-continuous function with range [0, 1]. This allows us to deal with the case where the distribution of the compensated variable is a step function, as would be the case if this variable is discrete. For any $(x_1, x_2) \in A_1 \times A_2$:

$$F(x_1, x_2) = \int_{[0,x_2]} F_1^2(x_1 | X_2 = t)\, dF_2(t). \tag{1}$$

Let $U(x_1, x_2)$ be the utility function, which is assumed to be twice continuously differentiable with respect to x_1 and x_2. The partial derivatives of U with respect with each variable are denoted with subscripts and are calculated in the usual manner. Our approach is to assume that the differentiability of U is independent of the actual support of the distribution F, i.e. the possible values of X_1 and X_2. This assumption mostly matters for X_2 since we consider the possibility that the distribution of the compensated variable is discrete, for example, if X_2 is household size or a discrete measure of health status. This could be interpreted as a model for an unobserved latent continuous variable $\tilde{X}_2$ for X_2. For the latent model, with a continuous distribution of $\tilde{X}_2$, the utility $U(x_1, \tilde{x}_2)$ is differentiable as usual with respect to any realized level $\tilde{x}_2$ of $\tilde{X}_2$. The only additional assumption here is that $U(x_1, \tilde{x}_2) = U(x_1, x_2)$ when $\tilde{x}_2 = x_2$, i.e. for the possible discrete realizations of X_2. In the family size example, X_2 may stand for the number of children in the family whereas $\tilde{X}_2$ may represent the unobserved proportion of days in the year that children spend in the family. In case of divorced parents, this latent variable may be considered as continuous. The use of a latent model enables us to provide a convenient mathematical translation of intuitive normative conditions in the form of signs of derivatives of U.

The social welfare function associated with F is assumed to be additively separable and can be written as

$$W_F := \int_{A_1 \times A_2} U(x_1, x_2)\, dF(x_1, x_2).$$

Using the decomposition in (1), we obtain

$$W_F = \int_{A_2} \left[\int_{A_1} U(x_1, x_2)\, dF_1^2(x_1 | X_2 = x_2) \right] dF_2(x_2), \tag{2}$$

where the marginal distribution of X_2 appears explicitly. The expression in brackets in (2) is the social welfare of the subpopulation of individuals having in common the same level of attribute 2. Thus, total social welfare is the sum over x_2 of the welfare levels of these subpopulations. This expression for social welfare generalizes that of [4, Expression 12.3, p. 353] in which F_2 was assumed to be discrete and F_1^2 continuous.

Consider the inner integral in expression (2). $U(x_1, x_2)$ is a differentiable function over A_1, while F_1^2 is a strictly increasing, continuous function over A_1. Thus, neither function has a discontinuity point on A_1. Consequently, the classical formula of integration by parts applies. When integrating with respect to X_2, the classical formula also applies because the assumptions made about the utility function preclude situations where U and F_1^2 have a common point of discontinuity (see, for instance, Billingsley [8, Theorem 18.4, p. 240]).[5] Thus, these assumptions allow

[5] The theorem says: Let F and G be two non-decreasing, right-continuous functions on an interval $[a, b]$. If F and G have no common points of discontinuity in $(a, b]$, then the classical formula of integration by parts applies.

us to treat in a unified way both the case of a discrete and continuous distribution of the compensated variable.[6]

The change in social welfare between any two distributions F and F^* is given by

$$\Delta W_U := W_F - W_{F^*} = \int_{A_1 \times A_2} U(x_1, x_2)\, d\Delta F(x_1, x_2),$$

where ΔF denotes $F - F^*$.

As typically done in the social welfare literature, we define social welfare dominance as unanimity over a family of social welfare functions defined by a given set of utility functions.

Definition. F dominates F^* for a family U of utility functions if and only if $\Delta W_U \geqslant 0$ for all utility functions U in U. This is denoted by $F D_{\mathcal{U}} F^*$.

2.1. Our main class

We start with the set $\mathcal{U}^2$ of increasing utility functions that satisfy the following signs for the partial derivatives:

$$\mathcal{U}^2 = \{U_1, U_2 \geqslant 0, U_{11} \leqslant 0, U_{22} \leqslant 0, U_{12} \leqslant 0, U_{121} \geqslant 0\}.^{7} \tag{3}$$

Non-decreasingness and concavity with respect to the compensating variable do not need to be justified. More care is required with the signs of the own first and second partial derivatives with respect to the compensated variable. When this is a "good" measured cardinally, such as a scale of good health, it seems natural to assume that utility levels are enhanced by better health and to introduce some concern for health inequality. On the other hand, when one wishes to consider needs described by a second variable, they entail a marginal disutility which violates the assumptions required for $\mathcal{U}^2$. However, it is straightforward to define a corresponding good as equal to the deviation of the needs variable with respect to its upper bound a_2. Including this good in the utility function delivers the right signs of the first partial derivatives for the class $\mathcal{U}^2$. In addition, $U_{22} \leqslant 0$ means that the disutility of the needs increases at a decreasing rate. Alternatively, one may want to consider situations in which the marginal burden of an additional child is increasing.

The two remaining signs ($U_{12} \leqslant 0$, $U_{121} \geqslant 0$) can be justified either by resorting to normative arguments that a social planner may adopt or by relying on traditional welfare analysis where social welfare is expressed as a sum of individual utility functions. We provide a general argument based on compensation principles for the first interpretation and illustrate the second by means of two examples.

We capture the idea that compensation is good for social welfare by imposing a negative sign on the cross-derivative of the utility function with respect to the compensating and compensated variables. In other words, the marginal utility of income is assumed to be non-increasing in the

[6] These assumptions do not cover the case where the utility is only defined for the actual support of the variables. In that case, common points of discontinuity may arise and correction terms have to be introduced when integrating by parts. Some of the problems identified by Fishburn and Lavalle [17] on unidimensional grids can then emerge.

[7] The class $\mathcal{U}^1$ will be introduced later in the text.

level of the compensated variable. For instance, the healthier an individual is, the lower are her claims to income redistribution, other things being equal.

Moreover, applying compensation seems all the more appropriate given that handicapped or needy people often belong to the lower tail of the income distribution. Rich persons who are ill may be thought to deserve less compensation than poor sick persons. We incorporate such considerations through a second compensation assumption which involves the asymmetric treatment of attributes.[8] Namely, we assume that the reduction in marginal utility of income with the level of the compensated variable is non-increasing in the agent's income. In this case, the gap in marginal utilities of income between a rich healthy and a rich ill person is not larger than between a poor healthy and a poor sick person.

A reader familiar with the literature will recognize that these assumptions are akin to those made by AB2 in a context where the compensated variable is discrete. In other words, our analysis introduces AB2 assumptions into the AB1 framework. To the best of our knowledge, such an approach has not been pursued so far.

We now discuss two examples which illustrate (1) the negativity of the second-order partial cross-derivatives and (2) the other signs of the partial derivatives, including that of U_{121}, as reasonable requirements for the social welfare evaluation explicitly described as a sum of individual utilities corresponding to different family sizes.

Example 1 *(Household size).* The second attribute is the deviation of household size n from some maximal benchmark $\bar{n}$, i.e., $x_2 = \bar{n} - n$, while the first attribute is household income, y. Here, $\bar{n} - n$ can be seen as representing the degree of satisfaction of household needs. We now examine the conditions for a household utility function $U(x_1, x_2)$ to belong to $\mathcal{U}^2$, where family size is treated as a continuous numerical variable for convenience. This corresponds to conditions ensuring that, with a slight abuse of notation, the function $U(y, n)$ satisfies: $U_y \geqslant 0$, $U_{yy} \leqslant 0$, $U_n \leqslant 0$, $U_{nn} \leqslant 0$, $U_{yn} \geqslant 0$ and $U_{yny} \leqslant 0$.

One popular way of dealing with needs expressed in terms of household size is to specify an equivalence scale function, $e(n)$. Then, social welfare is equal to the sum, over the population, of the utility levels of equivalent incomes. These equivalent incomes are defined as $\frac{y}{e(n)}$. In this setting, Ebert [15] proposed $U(y, n) = e(n)v(\frac{y}{e(n)})$ with $e'(n) \geqslant 0$. Assuming $v' \geqslant 0$ and $v'' \leqslant 0$, this specification ensures that $U_y \geqslant 0$, $U_{yy} \leqslant 0$ and $U_{yn} \geqslant 0$.

Imposing $U_{yny} \leqslant 0$ requires that the elasticity of v'' with respect to equivalent income have to be greater than 1 in absolute terms. For example, a function which is isoelastic with respect to income, $v(x) = \frac{1}{1-\beta}x^{1-\beta}$, with $0 \leqslant \beta < 1$, satisfies this condition. $U_n = e'(n)(v(\frac{y}{e(n)}))(1 - \epsilon)$, where $\varepsilon = \frac{v'(\frac{y}{e(n)})}{v(\frac{y}{e(n)})}\frac{y}{e(n)}$ is the elasticity of v with respect to equivalent income. Assuming that either v is negative and $\epsilon \leqslant 1$, or v is positive and $\epsilon \geqslant 1$, produces the required sign: an additional family member is a bad. Finally, $U_{nn} = e''(n)v(\frac{y}{e(n)})(1 - \epsilon) + \frac{(e'(n))^2 y^2}{(e(n))^3}v''(\frac{y}{e(n)})$. A linear equivalence scale ensures that this derivative is negative. Another favorable case is that of an isoelastic utility $v(x)$ and $e(n) = n^\theta$ with $0 < \theta \leqslant 1$, as in Banks and Johnson [6]. Then, $U_{nn} \leqslant 0$ if and only if $\theta\beta \leqslant 1$, which is ensured by the assumptions made about β and θ.

[8] Trannoy [37] describes elementary transformations leading to such a restriction on the utility function. Muller and Trannoy [30] discuss in detail how the signs of cross-partial derivatives are introduced in the literature, sometimes on the grounds of correlation-increasing majorization.

Finally, in this specific example of an equivalence scale, $U_{yynn} = (-\beta)y^{-\beta-1}(\theta\beta)(\theta\beta - 1)n^{\theta\beta-2}$. Then, if $\theta\beta \leqslant 1$, the sign of U_{nnyy} is the opposite of what is assumed in the second class considered by AB1. In this case, the example is in the set $\mathcal{U}^2$ but not in the smaller set $\mathcal{U}^{AB1_2}$ (see Table 1 below). Thus, this example provides support for considering the set $\mathcal{U}^2$ without making the further assumptions that lead to $\mathcal{U}^{AB1_2}$.

Example 2 *(Indirect household utility).* The equivalence scale is a kind of reduced form and one may prefer a more structural framework with which to represent budget sharing among household members. Bourguignon [9] investigated the properties of the household indirect utility function for the unitary model of households with public goods. In this model, each individual has the same continuous, increasing and strictly concave utility function V defined on two attributes: the private consumption level x and a within-household public good of consumption level g. To simplify, we assume that each household of size n behaves as a utilitarian society and that all prices it faces are unity.[9] Household income y is allocated according to the following decision rule: $\max_{x,g} nV(x, g)$ subject to $nx + g = y$, where each person in the family gets x. The corresponding first-order condition is: $n(V_x - nV_g) = 0$, the solution of which is x^*, the optimal private consumption. The indirect utility function can be calculated by substituting the demand functions $x(y, n)$ and $g(y, n)$ into the direct utility function: $U(y, n) = nV(x(y, n), y - nx(y, n))$.[10] As in the first example, family size is a bad; i.e., $U_n = V - nx^*V_g \leqslant 0$. We now establish the set of properties which ensure the same signs for the partial derivative as in Example 1.

We first make three assumptions: (a) The private good is normal, $x_y^* \geqslant 0$, which, using the strict quasi-concavity of V, requires that $-V_{xg} + nV_{gg} \leqslant 0$. It turns out that this condition implies that $x_n^* \leqslant 0$ as well.

(b) The public good is normal. We then deduce that $1 - nx_y^* \geqslant 0$ from fully differentiating the budget constraint with respect to y.

(c) Private consumption and the public good are substitutes ($V_{xg} \leqslant 0$).

Using the envelope theorem, we obtain $U_y = nV_g \geqslant 0$ and $U_{yy} = n(V_{xg}x_y^* + V_{gg}(1 - nx_y^*)) \leqslant 0$. We now deal with the conditions on U_{nn} and U_{yn}. We have

$$U_{nn} = -x^*(2 + \eta_n)V_g - nx^*x_n^*V_{xg} + n\left(x^*\right)^2(1 + \eta_n)V_{gg},$$

where $\eta_n = \frac{nx_n^*}{x^*}$ is the elasticity of the individual private consumption with respect to family size. We also have

$$U_{yn} = V_g + x^*\eta_n V_{xg} - nV_{gg}x^*(1 + \eta_n).$$

If in addition, we assume (d) $\eta_n > -1$, then $U_{nn} \leqslant 0$ and $U_{yn} \geqslant 0$. Therefore, provided the proportional increase in individual consumption is smaller than the proportional reduction in household size, a reasonable conjecture, we obtain all the required signs for the first-order and second-order partial derivatives.

[9] The same model holds if one only assumes that households allocate goods efficiently. For the problem at hand, it is simpler to consider that individuals (with identical utility functions) are treated symmetrically.

[10] Note that the indirect utility is here denoted by U, while the direct utility is denoted by V. This is in accordance with the use of U in the social welfare objective.

Table 1
Classes of relevant utility functions for the bivariate case.

Acronym	Authors	1st degree	2nd degree	3rd degree	4th degree
$\mathcal{U}^{AB1_1}$	$AB1_1$	$U_1, U_2 \geqslant 0$	$U_{12} \leqslant 0$		
$\mathcal{U}^{GM}$	Gravel and Moyes	$U_1, U_2 \geqslant 0$	$U_{11}, U_{12} \leqslant 0$		
$\mathcal{U}^{IDIC_2}$	$IDIC_2$	$U_1, U_2 \geqslant 0$	$U_{11}, U_{22}, U_{12} \leqslant 0$		
$\mathcal{U}^1$	Bazen and Moyes	$U_1, U_2 \geqslant 0$	$U_{11}, U_{12} \leqslant 0$	$U_{121} \geqslant 0$	
$\mathcal{U}^2$	Our main class	$U_1, U_2 \geqslant 0$	$U_{11}, U_{22}, U_{12} \leqslant 0$	$U_{121} \geqslant 0$	
$\mathcal{U}^{AB1_2}$	$AB1_2$	$U_1, U_2 \geqslant 0$	$U_{11}, U_{22}, U_{12} \leqslant 0$	$U_{121}, U_{212} \geqslant 0$	$U_{1122} \leqslant 0$

Finally, we investigate the conditions under which we can obtain the required sign for U_{yny}. In the simple case in which all other third-order derivatives can be neglected, we obtain

$$\begin{aligned} U_{yyn} &= V_{xg}x_y^* + V_{gg}\big(1 - nx_y^*\big) + nV_{xg}x_{yn}^* - nV_{gg}\big(x_y^* + nx_{yn}^*\big) \\ &= \big(x_y^* + nx_{yn}^*\big)(V_{xg} - nV_{gg}) + V_{gg}\big(1 - nx_y^*\big). \end{aligned}$$

If we further assume that (e) $g_{yn} \geqslant 0$, that is, when households become richer, consumption of the public good increases with family size (e.g., as income increases, housing and durable goods become more important and food consumption less important), then $x_y^* + nx_{yn}^* \leqslant 0$. Assumptions (a), (b) and (e) allow us to conclude that $U_{yyn} \leqslant 0$. Therefore, the signs of the partial derivatives corresponding to $\mathcal{U}^2$ can be recovered for natural specifications of the preferences in this example.

2.2. *Overview of the classes of utility functions*

It is useful to compare our class of utility functions to those which have been studied in the economic literature and the stochastic order literature for the bivariate case. Table 1 presents the classes of utility functions that are relevant for the discussion in this paper. These classes are defined by the signs of their partial derivatives. We examine different classes beginning from the least discriminating.

We start with the first class of functions in Atkinson and Bourguignon [3], which is close to those leading to first-degree stochastic dominance since concavity with respect to income is not even required. We finish with their second class, which can be viewed as a maximalist class with required signs of partial derivatives up to the fourth order. In the latter case, together with $U_{12} = U_{21} \leqslant 0$, the sign condition for U_{212} is in line with the idea that the second attribute can also be used to compensate for a deficiency in the first attribute to both the first and second degrees. Note that two elements of the so-called 'increasing concave order' in the literature on multidimensional stochastic orders correspond to the two classes considered by Atkinson and Bourguignon [3].[11]

While $\mathcal{U}^{AB1_1}$ or $\mathcal{U}^{AB1_2}$ treats the two attributes symmetrically, Gravel and Moyes [23] and Bazen and Moyes [7] define two intermediate classes that require marginal utility to be decreasing with respect to the compensated variable only. In fact, when the compensated variable has solely an ordinal signification, it is meaningless to impose the monotonicity of its marginal utility.

[11] See for example Denuit, Lefèvre and Mesfioui [12], Denuit and Mesfioui [13] and Denuit, Eeckhoudt, Tsetlin and Walker [11].

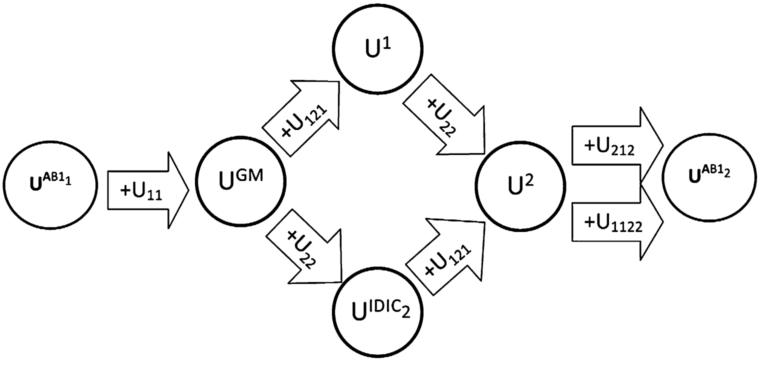

Fig. 1. The relevant classes of utility functions for bivariate dominance.

In that case, another type of asymmetric treatment of variables is introduced. Thus, the Gravel–Moyes and Bazen–Moyes criteria can be viewed as intermediate cases between $AB1_1$ and our class $\mathcal{U}^2$.

It is also possible to compare $\mathcal{U}^2$ with the so-called *increasing directionally concave order of degree s* (denoted by IDIC_s) class.[12] For the bivariate case, the (s)-*increasing directionally concave order* is defined by the dominance of all functions g such that $(-1)^{k_1+k_2+1}\frac{\partial^{k_1+k_2}}{\partial x_1^{k_1} x_2^{k_2}} g \geqslant 0$, for all non-negative integers $k_1 + k_2$ such that $1 \leqslant k_1 + k_2 \leqslant s$, where s is a non-negative integer greater than or equal to 2. The increasing directionally concave order of degree 2 corresponds to dominance for functions U such that $U_1, U_2 \geqslant 0$ and $U_{11}, U_{22}, U_{12} \leqslant 0$.

Fig. 1 illustrates how the different classes of utility functions can be ranked, starting from the broadest class $AB1_1$ up to the most discriminating class $AB1_2$. It illustrates the two paths followed by the approaches in the literature. However, this figure is not completely helpful for finding candidates for necessary and sufficient conditions for $\mathcal{U}^2$. In particular, while necessary and sufficient conditions are known for the Bazen and Moyes class, this is not the case for IDIC_2.[13]

The classes of utility functions in the needs approach correspond to those in the multidimensional approach with an additional restriction. The marginal distribution of needs is considered as fixed in the original needs approach. Consequently, it is not necessary to define the signs of the own partial derivatives with respect to the compensated attribute. With this restriction in mind, the signs of the partial derivatives for the Bourguignon [9] class corresponds to the Gravel–Moyes class. Moreover, the Atkinson and Bourguignon [4] class is included in the Bazen–Moyes class and in our class $\mathcal{U}^2$.

3. Stochastic dominance results

In this section, we state and discuss our general results in terms of second-degree stochastic dominance. It is convenient to define the two standard univariate second-degree stochastic

[12] See Denuit, Lefèvre and Mesfioui [12] and Denuit and Mesfioui [13].

[13] Note that Lemma 4.1 in combination with Proposition 3.1. in Denuit and Mesfioui [13] does not result in tractable conditions in the bivariate case. See more on this in footnote 15.

dominance terms[14]

$$H_i(x_i) = \int_0^{x_i} F_i(s)\,ds, \quad i = 1, 2,$$

the bivariate term

$$H(x_1, x_2) = \int_0^{x_1}\int_0^{x_2} F(s, t)\,ds\,dt,$$

and the term where we integrate the distribution function only with respect to the compensating variable

$$H_1(x_1; x_2) = \int_0^{x_1} F(s, x_2)\,ds.$$

Since $H_1(x_1; x_2) = \int_0^{x_2}\int_0^{x_1} F_1^2(s|X_2 = x_2)\,ds\,dF_2(x_2)$, it can also be seen as arising from cumulating the conditional distribution F_1^2. For the sake of completeness, we recall the results obtained by AB1.

Theorem 1. *(See Atkinson and Bourguignon [3].) Let F and F^* be two cdfs.*

(a) $F D_{\mathcal{U}^{AB1_1}} F^*$ *if and only if* $\forall x_1 \in A_1$, $\forall x_2 \in A_2$, $\Delta F(x_1, x_2) \leqslant 0$.
(b) $F D_{\mathcal{U}^{AB1_2}} F^*$ *if and only if* $\forall x_1 \in A_1$, $\forall x_2 \in A_2$, $\Delta H_1(x_1) \leqslant 0$, $\Delta H_2(x_2) \leqslant 0$ *and* $\Delta H(x_1, x_2) \leqslant 0$.

In the case of the set of utility functions $\mathcal{U}^2$, we obtain the following result.

Proposition 1. *Let F and F^* be two cdfs. If*

$$\forall x_2 \in A_2, \quad \Delta H_2(x_2) \leqslant 0 \tag{B}$$

and

$$\forall x_2 \in A_2,\ \forall x_1 \in A_1, \quad \Delta H_1(x_1; x_2) \leqslant 0, \tag{C}$$

then $F D_{\mathcal{U}^2} F^*$.

Condition (B) is the typical second-degree stochastic dominance expression applied to the second attribute. Condition (C) involves a mixed second-degree stochastic dominance term, which is the cdf of the *joint distribution* integrated once with respect to the first attribute. In particular, Condition (C) implies second-degree stochastic dominance for the first attribute since $H_1(x_1; a_2) = H_1(x_1)$.

[14] The letter H indicates that F is integrated once with respect to a variable. The index variable is denoted by a subscript. The semi-colon in $H_1(x_1; x_2)$ indicates that the variable on the left-hand side of the semi-colon is used for integration one more time than the variable on the right-hand side. A comma between the two variables indicates that they are used for the same number of integrations.

A more intuitive condition than Condition (C) can be derived from the equivalent poverty ordering. Foster and Shorrocks [21] showed that unidimensional stochastic dominance tests are equivalent to unidimensional poverty orderings. Introducing an absolute poverty line for the second attribute, z_2, we can define the absolute poverty gap term for this attribute:

$$P_2(z_2) = \int_{[0,z_2]} (z_2 - x_2)\, dF(x_2),$$

which can be used to generate an equivalent condition to Condition (B) in terms of absolute poverty gaps. Moreover, introducing the absolute poverty line, z_1, in the first attribute, and defining

$$P_1(z_1; x_2) = \int_{[0,x_2]} \int_{[0,z_1]} (z_1 - x_1)\, dF(x_1, t_2),$$

we obtain the absolute poverty gap of the compensating variable for the population below a level z_1 of this variable and below a level x_2 of the compensated variable. This expression turns out to be equivalent to the mixed second-degree stochastic dominance term in Condition (C). Using these two poverty gaps allows us to rewrite Proposition 1 in a fashion more convenient for applied work.

Corollary 1. *Let F and F^* be two cdfs.*

$$\left[\Delta P_2(z_2) \leqslant 0,\ \forall z_2 \in A_2 \text{ and } \Delta P_1(z_1; x_2) \leqslant 0,\ \forall z_1 \in A_1 \text{ and } \forall x_2 \in A_2\right] \\ \Rightarrow \quad F D_{\mathcal{U}^2} F^*.$$

The following remark may be useful to economists dealing with attributes which reduce welfare, as in the two examples.

Remark. Consider the companion class of $\mathcal{U}^2$, $\mathcal{U}^{2*} = \{U_1 \geqslant 0, U_2 \leqslant 0, U_{11} \leqslant 0, U_{22} \leqslant 0, U_{12} \geqslant 0, U_{121} \leqslant 0\}$, where the second attribute is a bad with a disutility which is increasing and concave. Then, it is sufficient to change Condition (B) to $\Delta H_2(x_2) \geqslant 0$ in Proposition 1 to obtain a sufficient dominance result for the $\mathcal{U}^{2*}$ class. When the distribution of needs is invariant between the two situations being compared, this condition vanishes.

We attempted to find fully necessary conditions using the counter-example approach introduced by Fishburn and Vickson [18], that is, by specifying a utility function U in $\mathcal{U}^2$ such that $\Delta W_U < 0$ when either Condition (B) or (C) is violated. Condition (B) is necessary for dominance as a consequence of standard results for unidimensional stochastic dominance. In fact, setting $U_1 = 0$, one can consider the set of all utility functions in $\mathcal{U}^2$ that are twice differentiable, increasing and concave in the second argument. Condition (C) evaluated at the upper bound a_2 is also necessary since it is the second-order stochastic dominance condition applied to the first attribute.

The difficulty with the proof of necessity arises when dealing with Condition (C). Our approach was to try to construct a counter-example by supposing that $\Delta H_1(x_1; x_2) > 0$ in a small neighborhood of a given point (x_1^*, x_2^*). Then, a function U is characterized by its higher partial derivative U_{121} specified as a constant in this neighborhood and zero elsewhere. Finally, U is integrated out successively within and outside the neighborhood. An appropriate choice of neighborhood and of the constant implies that $\Delta W_U = -\Delta H_1(x_1^*; x_2^*)$, which was intended to provide the counter-example. Unfortunately, this constructive approach is hampered by the necessity of globally satisfying all the restrictions in $\mathcal{U}^2$. It turns out that these restrictions cannot be imposed simultaneously over the whole two-dimensional domain as they are geometrically incompatible.

As mentioned earlier, the traditional approach to necessary conditions is based on counter-examples, which are generator functions of the continuous convex sets of utilities (or 'test functions' as in Athey [1]). Minimal and maximal sets of generators are studied in Müller [29]. Our class is clearly convex since convex combinations of functions preserve the signs of the derivatives involved. However, it is quite possible that no such generator system exists for an infinite dimensional set of functions like $\mathcal{U}^2$ and, furthermore, if one exists, it may not be straightforward to find it. In particular, functions proposed in the literature for other classes which we have considered are not relevant in our case since they do not satisfy the conditions to be in $\mathcal{U}^2$. For example, certain functions in the closure of the convex combinations of the popular counter-example functions $U^*(x_1, x_2) = -\max\{0, z_1 - x_1\}I[0, z_2]$, where $z_1 \in [0, a_1]$ and $z_2 \in [0, a_2]$ and I is the indicator function and $U^{**}(x_1, x_2) = -\max\{0, z_2 - x_2\}$, where $s_2 \in [0, a_2]$, do not satisfy $U_{22} \leqslant 0$. This is unfortunate since using these functions would yield the stochastic dominance conditions in Proposition 1. The difficulty in finding generator functions is further emphasized by the fact that they are not yet available in the technical literature for the class IDIC_2, a symmetric class close to our class $\mathcal{U}^2$.[15]

These technical difficulties do not arise with the set $\mathcal{U}^1$ considered by Moyes [28] and Bazen and Moyes [7]. In this case, the marginal utility with respect to the compensated variable is not required to be decreasing, which relaxes a crucial restriction on the counter-example restriction. Dominance in $\mathcal{U}^1$ corresponds to dominance for the functions $U^{***}(x_1, x_2) = -I[0, s_2]$, where $s_2 \in [0, a_2]$, and $U^*(x_1, x_2)$. In this case, replacing Condition (B) in our results with $\Delta F_2(x_2) \leqslant 0$ (the first-order unidimensional stochastic dominance condition) yields a necessary and sufficient condition for $\mathcal{U}^1$.

4. Inverse stochastic dominance

Expressing conditions of stochastic dominance in terms of Lorenz curves is an attractive approach to inequality measurement, as Atkinson showed more than thirty years ago (Atkinson [2]). It is also straightforward with our standard expression for second-order stochastic dominance (B) in Proposition 1. We now translate Condition (C) into an inverse stochastic dominance expression.

We first return to the formal definition of the Generalized Lorenz curve. The right-inverse of a positive non-decreasing and right-continuous function $F(x)$ is defined by: $F^{-1}(p) = \sup_{F(x) \leqslant p} x$, with p in [0, 1]. The Generalized Lorenz (GL) curve of the marginal cdfs F_i for $i = 1, 2$, denoted by $\mathcal{L}_{F_i}(p)$ is defined on [0, 1] by $\mathcal{L}_{F_i}(p) = \int_0^p F_i^{-1}(t)\,dt$. In the cases in which

[15] Generators are provided by Denuit and Mesfioui [13] in Lemma 4.1 for all orders of dominance IDIC_s of index $s \geqslant n$, where n is the number of variables considered. Our case of interest is $s = n = 2$. However, this lemma cannot be used for $s = n = 2$ because the Taylor expansion from which the generators are obtained collapses. Technically, this is because the generators of IDIC_2 are characterized as the intersection of the generators of $(1, 1)$-increasing concave, $(2, 0)$-increasing concave and $(0, 2)$-increasing concave classes (for the definitions of these classes, see Denuit and Mesfioui [13]). But the Taylor expansions collapse for the last two classes. This point has been confirmed in a correspondence with Professor Michel Denuit.

the first attribute is income, the Generalized Lorenz curve of F_1 is based on the cumulative total income received by the poorest proportion p of the population. In this representation, individuals are ranked according to their income.

We now define a related concept: the Projected Generalized Lorenz (PGL) curve for a given value of x_2. For each value x_2 in X_2, the *projected distribution function* of x_1, F_{x_2}, is defined on X_1 by the equation $F_{x_2}(x_1) = F(x_1, x_2)$. It is obtained by a projection of the joint distribution $F(x_1, x_2)$ onto the Cartesian plane going through x_2. This value may be viewed as a threshold below which the compensation is deemed as favorable from a normative point of view. For a given x_2, $F_{x_2}(x_1)$ is at most equal to $F(a_1, x_2) = F_2(x_2)$. Due to our hypotheses concerning F_1 and F_1^2, F_{x_2} is continuous and strictly increasing.

The Projected Generalized Lorenz curve for a given value of x_2 is the Generalized Lorenz curve for the projected distribution function for x_2. The corresponding right inverse is defined by setting, $\forall p \in [0, F_2(x_2)]$, $F_{x_2}^{-1}(p) = \sup_{F_{x_2}(x_1) \leqslant p} x_1$.

Definition. The Projected Generalized Lorenz (PGL) curve on $[0, F_2(x_2)]$, for a given value of x_2, is defined by $\mathcal{C}_{F_{x_2}}(p) = \int_0^p F_{x_2}^{-1}(t)\,dt$.

In the cases in which the first attribute is income, the quantity $\mathcal{C}_{F_{x_2}}(p)$ is the cumulative income received by the poorest proportion p of the population having at most a level x_2 of the compensated variable. The Projected Generalized Lorenz curve $\mathcal{C}_{F_{x_2}}(p)$ is related by a scale factor $1/F_2(x_2)$ to the Generalized Conditional Lorenz curve for x_1, defined using the conditional distribution of x_1.[16] With this definition, we obtain the following result.

Proposition 2. *Let F and F^* be two cdfs.*

$$\forall x_2 \in A_2, \quad \big[\Delta H_1(x_1; x_2) \leqslant 0,\ \forall x_1 \in A_1 \\ \Rightarrow \mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F^*_{x_2}}(p),\ \forall p \in \big[0, \min\big(F_2(x_2), F_2^*(x_2)\big)\big]\big] \tag{L_1}$$

$\forall x_2 \in A_2$ such that $\Delta F_2(x_2) \leqslant 0$,

$$\big[\Delta H_1(x_1; x_2) \leqslant 0,\ \forall x_1 \in A_1 \Leftrightarrow \mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F^*_{x_2}}(p),\ \forall p \in \big[0, F_2(x_2)\big]\big] \tag{L_2}$$

Moreover, the above inequalities between the PGL curves in (L_1) are strict at all points x_1 for which the corresponding projected distribution functions do not intersect.

In words, the first statement says that a necessary condition for Condition (C) of Proposition 1 to hold is that the Projected Generalized Lorenz curve of the dominant distribution must be above that of the dominated distribution. The information lies in the domain condition. For a given value of the compensated variable x_2, this statement only has to be true for all proportions of the population up to the minimum of $F_2(x_2)$ and $F_2^*(x_2)$. In the first statement, it may be the case that the distribution of the compensated variable for the dominant distribution does not dominate at the first order that for the dominated distribution. If we assume that this first-order stochastic

[16] Indeed, for a given value of x_2 and for any $x_1 \in A_1$, the conditional cdf is defined by $G_{x_2}(x_1) = \frac{F_{x_2}(x_1)}{F_2(x_2)} = F_1(x_1 | X_2 \leqslant x_2)$. For any $p \in [0, 1]$, its inverse $G_{x_2}^{-1}(p) = \sup_{G_{x_2}(x_1) \leqslant p} x_1$. Thus, the GL curve corresponding to the subpopulation for which the level of the compensated variable is at most x_2 is defined by $\mathcal{L}_{F_{x_2}}(p) = \int_0^p G_{x_2}^{-1}(t)\,dt$, $\forall p \in [0, 1]$. Dividing by $F_2(x_2)$ introduces the same change in the analysis as in inequality analysis when dividing by the mean and using Lorenz curves instead of GL curves.

dominance holds, then we get the equivalence between stochastic dominance Condition (C) and the inverse stochastic dominance condition in terms of the Projected Generalized Lorenz curves. This is the substance of statement (L_2).

In statements (L_1) and (L_2), PGL dominance is imposed for various ranges of the proportion p. These ranges are non-decreasing in x_2. As a consequence, (L_1) or (L_2) dominance conditions are not very demanding for levels of x_2 lying in the lower tail. In contrast, they become increasingly demanding, as is the case with the Lorenz dominance condition, when the level of x_2 approaches the upper bound of its distribution support.

In statement (L_1), the dominance of the PGL curve of the compensating variable for any value of the compensated variable is necessary over a domain that is determined by the intersection of the supports of the two compared PGL curves. It turns out that the PGL test is sufficient only when the distribution of the compensated variable for F dominates its counterpart for F^* to the first order, which is in particular the case when the marginal distribution of the compensated variable is fixed. As a consequence, for the family of utility functions $\mathcal{U}^1$ considered by Moyes [28] and Bazen and Moyes [7], we obtain a complete characterization result based on the PGL curve.

Corollary 2. *Let F and F^* be two cdfs. $F D_{\mathcal{U}^1} F^*$ if and only if $\Delta F_2(x_2) \leqslant 0$, $\forall x_2 \in A_2$ and $\mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F^*_{x_2}}(p)$, $\forall p \in [0, F_2(x_2)]$, $\forall x_2 \in A_2$.*

In situations where a first-order stochastic dominance relation does not hold in comparing the distributions of the compensated variable, there exist values of x_2 where the proportion of individuals having at most this level is higher in F than in F^*. Condition (B) in Proposition 1 implies that this cannot occur for the smallest level of x_2. It can also be observed that

$$\begin{aligned}
&\forall x_2 \in A_2 \quad \text{such that} \quad F_2^*(x_2) \leqslant F_2(x_2),\\
&\quad \left[\Delta H_1(x_1; x_2) \leqslant 0,\ \forall x_1 \in \left[0, F_2^{-1}\left(F_2^*(x_2)\right)\right]\right]\\
&\qquad \Leftrightarrow \quad \left[\mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F^*_{x_2}}(p),\ \forall p \in \left[0, F_2^*(x_2)\right]\right].
\end{aligned}$$

So, roughly speaking, checking the PGL condition is necessary and sufficient at the bottom of the joint distribution, but not at the top.

In the discrete case, the PGL tests in (L_1) or (L_2) can be performed sequentially. One starts by considering the observations corresponding to the lowest level of the compensated variable; then, one adds to this sample the observations corresponding to the second lowest level and so on.

For the sake of illustration, consider the case of a discrete realization for the compensated variable. Specifically, let F_2 and F_2^* be two step functions with jumps at $x_{21}, \ldots, x_{2k}$ for F_2 (respectively $x_{21}^*, \ldots, x_{2l}^*$ for F_2^*). Statement (L_1) indicates that no condition on ΔF_2 is required for any level of x_2 strictly below $\max(x_{21}, x_{21}^*)$. Indeed, the range of p considered in statement (L_1) is $[0, \min(F_2(x_2), F_2^*(x_2))]$ and in this case $\min(F_2(x_2), F_2^*(x_2)) = 0$. The first comparison in the sequence starts at $x_2 = \max(x_{21}, x_{21}^*)$. The PGL curves are compared for the subpopulation of individuals with a compensated attribute lower than or equal to a given level x_2. This is illustrated in Fig. 2. The PGL curve of the dominating distribution must be above the PGL curve of the dominated distribution for the poorest proportions of population according to the compensated variable up to the minimum of $F_2(x_2)$ and $F_2^*(x_2)$.

If the first sequential comparison yields a positive dominance result, then the PGL test is repeated for the next level of x_2 strictly larger than $\max(x_{21}, x_{21}^*)$ in the common support of $F_2(x_2)$ and $F_2^*(x_2)$. If the result is again positive, an additional iteration is carried out using the

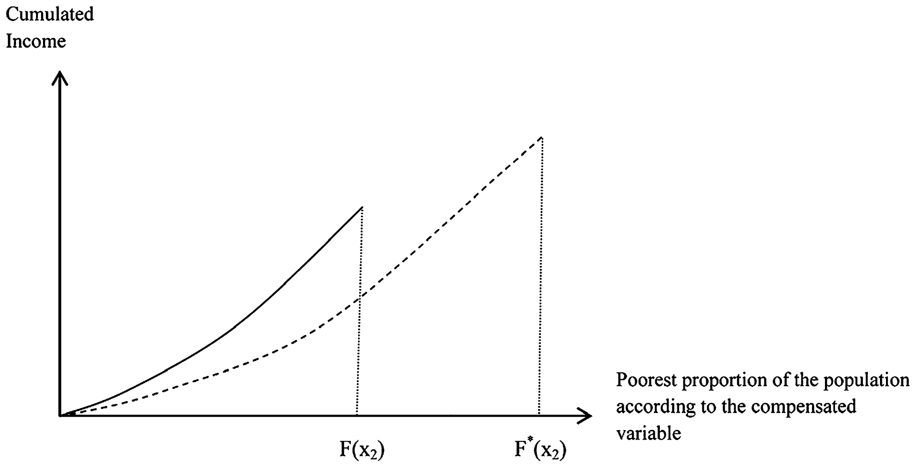

Fig. 2. Comparison of Projected Generalized Lorenz Curves: The solid line (resp. the dotted line) is the PGL curve of the F (resp. F^*) distribution.

next level of x_2 and so on, up to the last sequential comparison, which occurs for the maximum level in the two distributions F_2 and F_2^*. In this case, the comparison is tantamount to performing the classic GL test, since at this level of x_2, F_2 and F_2^* both take the value 1, the range of p is $[0, 1]$, $F(x_1, x_2) = F(x_1)$ and $F^*(x_1, x_2) = F^*(x_1)$.

The relationship between the PGL criterion and the SGL criterion pioneered by AB2 is simpler in the case of identical marginal distributions of the compensated variable. In this case, the PGL curve corresponds to the 'conditional GL curve', i.e., the GL curve of the conditional distribution of x_1 given x_2. Thus, in the case of identical marginal distributions of the compensated variable, our criterion boils down to the SGL criterion and we are back to the needs approach considered by AB2. In this particular setting, we have thus extended their results to the case of a continuous distribution of needs, as stated in the following corollary.

Corollary 3. *Let F and F^* be two joint cdfs such that $F_2 \equiv F_2^*$. Then, $F D_{\mathcal{U}^{AB2}} F^*$ if and only if $\mathcal{L}_{F_{x_2}}(p) \geqslant \mathcal{L}_{F_{x_2}}(p)$, $\forall p \in [0, 1]$, $\forall x_2 \in A_2$.*

5. Introducing transfer sensitivity

There has been some interest in the literature in the non-negativity of the own third-order partial derivatives for income ($U_{111} \geqslant 0$). This condition is related to the transfer sensitivity property, which implies that the social planner is more sensitive to income transfers performed at the bottom of the income distribution than at the top.[17] One may also be interested in imposing transfer sensitivity for the compensated variable ($U_{222} \geqslant 0$) rather than for the compensating variable. In that case, we obtain a refinement of our sufficient conditions for the following set of utility functions:

$$\mathcal{U}^3 = \{U_1, U_2 \geqslant 0, U_{11} \leqslant 0, U_{22} \leqslant 0, U_{12} \leqslant 0, U_{222} \geqslant 0, U_{121} \geqslant 0\}.$$

[17] See Shorrocks and Foster [36] for a general study of transfer sensitivity and Lambert and Ramos [27] for an application to the needs approach.

We define the third-order stochastic dominance term for the marginal distributions: $L_i(x_i) = \int_0^{x_i} H_i(s)\,ds$, $i = 1, 2$. For this case, we obtain the following proposition.[18]

Proposition 3. *Let F and F^* be two cdfs. If $\Delta H_2(a_2) \leqslant 0$, $\Delta H_1(x_1; x_2) \leqslant 0$, $\forall x_2 \in A_2$, $\forall x_1 \in A_1$ and $\Delta L_2(x_2) \leqslant 0$, $\forall x_2 \in A_2$, then $F D_{\mathcal{U}^3} F^*$.*

We can therefore achieve slightly more discriminating dominance conditions by incorporating the assumption of transfer sensitivity with respect to the compensated variable. The last condition in Proposition 3 is the unidimensional condition of third-order stochastic dominance applied to the compensated variable. It is supplemented by a terminal condition of second-order stochastic dominance that the mean of the compensated variable is larger for the dominant distribution than for the dominated distribution. By successively referring to the three families $\mathcal{U}^1$, $\mathcal{U}^2, \mathcal{U}^3$, which correspond respectively to first-, second- and third-order analysis of the compensated variable, we can then generate increasingly less demanding criteria and, thus, fewer partial quasi-orderings.

6. Conclusion

Using compensation principles, we have proposed an integrated treatment of the needs and of the multidimensional stochastic dominance approaches in the bivariate case. We have derived a stochastic dominance condition which generalizes the Sequential Generalized Lorenz criterion to a continuous distribution of needs.

We conclude with some general observations concerning the differences between the two approaches from theoretical and empirical perspectives. The multidimensional approach requires information about signs of marginal utilities of the compensated attributes and of their slopes. However, one may feel uncomfortable with restricting such signs for certain types of need. The example of family size is an illustration of this point. Should we treat a child as a cost or as a “good”? The examples presented in Section 2 are based on the assumption that a child is viewed as a cost, given a fixed household budget. However, one may instead prefer not to make such assumptions about demographic family structure on the grounds that they would not achieve unanimity among social scientists. In this case, adhering to the needs approach alone seems sensible. In contrast, the multidimensional approach could be preferred when (1) it is clear that each attribute should contribute directly to individual welfare, and (2) when one wants to assess the impact of a policy measure on all dimensions of welfare.

Various extensions of our analysis are possible. In particular, empirical studies of actual compensation mechanisms operating in society could aid in the specification of restrictions on utility functions to be used for stochastic dominance results. From a theoretical standpoint, understanding the profound reasons for not achieving necessary and sufficient conditions in the multidimensional setting for every normatively meaningful class of utility functions is still an avenue for further research. This would help social scientists to identify the limitations in the use of desirable normative properties on utility functions that arise from technical obstacles that are encountered for achieving necessity conditions. An open question is if these limitations also hold when considering discrete needs levels.

[18] Fishburn and Lavalle [17] pointed out that in the case of a variable that only takes discrete values, the third-order dominance condition is more stringent than the one in Proposition 3.

As to extensions to a larger number of attributes, an example of this may be found in Muller and Trannoy [30], where a generalization of the Human Development Index based on income, education and health indicators is considered.

Appendix A

A.1. Proof of Proposition 1

Let

$$W_F = \int_{A_2}\left[\int_{A_1} U(x_1,x_2)\,dF_1^2(x_1|X_2=x_2)\right]dF_2(x_2). \tag{4}$$

Integrating the inner integral by parts gives

$$\int_{A_1} U(x_1,x_2)\,dF_1^2(x_1|X_2=x_2) = U(a_1,x_2)F_1^2(a_1|X_2=x_2) - U(0,x_2)F_1^2(0|X_2=x_2) - \int_{A_1} U_1(x_1,x_2)F_1^2(x_1|X_2=x_2)\,dx_1.$$

Noting that cdfs vanish at zero and $F_1^2(a_1|X_2=x_2)=1$ since F_1^2 is a cdf, integrating the last term of the RHS of the above expression by parts once again with respect to x_1 and substituting in (4), we get

$$W_F = \int_{A_2} U(a_1,x_2)\,dF_2(x_2) - \int_{A_2}\left[U_1(a_1,x_2)\int_{A_1} F_1^2(x_1|X_2=x_2)\,dx_1\right]dF_2(x_2) + \int_{A_2}\left[\int_{A_1} U_{11}(x_1,x_2)\left(\int_0^{x_1} F_1^2(s|X_2=x_2)\,ds\right)dx_1\right]dF_2(x_2). \tag{5}$$

Integrating the first term of the RHS of the above expression by parts gives

$$\int_{A_2} U(a_1,x_2)\,dF_2(x_2) = U(a_1,a_2)F(a_1,a_2) - \int_{A_2} U_2(a_1,x_2)F(a_1,x_2)\,dx_2. \tag{6}$$

Integrating the second term of the RHS of (5) by parts with respect to x_2 gives

$$-U_1(a_1,a_2)\left[\int_{A_2}\left[\int_{A_1} F_1^2(x_1|X_2=x_2)\,dx_1\right]dF_2(x_2)\right] + \int_{A_2} U_{12}(a_1,x_2)\left[\int_0^{x_2}\left[\int_{A_1} F_1^2(x_1|X=t)\,dx_1\right]dF_2(t)\right]dx_2. \tag{7}$$

Using Fubini's theorem,

$$\left[\int_0^{x_2}\left[\int_{A_1} F_1^2(x_1|X_2=t)\,dx_1\right]dF_2(t)\right] = \int_{A_1}\left[\int_0^{x_2} F_1^2(x_1|X_2=t)\,dF_2(t)\right]dx_1$$
$$= \int_{A_1} F(x_1,x_2)\,dx_1.$$

Expression (7) now reduces to:

$$= -U_1(a_1,a_2)\int_{A_1} F_1(x_1)\,dx_1 + \int_{A_2} U_{12}(a_1,x_2)\left[\int_{A_1} F(x_1,x_2)\,dx_1\right]dx_2. \tag{8}$$

Similarly, integrating the third term of the RHS of (5) by parts with respect to x_2, we obtain

$$\int_{A_1} U_{11}(x_1,a_2)\left[\int_{A_2}\left[\int_0^{x_1} F_1^2(s|X_2=x_2)\,ds\right]dF_2(x_2)\right]dx_1$$
$$- \int_{A_2}\int_{A_1} U_{112}(x_1,x_2)\left[\int_0^{x_2}\left[\int_0^{x_1} F_1^2(s|X_2=t)\,ds\right]dF_2(t)\right]dx_1\,dx_2,$$

which reduces to

$$= \int_{A_1} U_{11}(x_1,a_2)\left[\int_0^{x_1} F_1(s)\,ds\right]dx_1 - \int_{A_2}\int_{A_1} U_{112}(x_1,x_2)\left[\int_{A_1} F(s,x_2)\,ds\right]dx_1\,dx_2. \tag{9}$$

Similar expressions to (6), (8) and (9) can be derived for W_{F^*}. Using $\Delta F(a_1,a_2)=0$ because $F(a_1,a_2)=1$ for any F, the difference in welfare is:

$$\Delta W_U = \int_{A_1\times A_2} U(x_1,x_2)\Delta dF(x_1,x_2)$$
$$= -\int_{A_2} U_2(a_1,x_2)\Delta F_2(x_2)\,dx_2 - U_1(a_1,a_2)\int_{A_1} \Delta F_1(x_1)\,dx_1$$
$$+ \int_{A_2} U_{12}(a_1,x_2)\left[\int_{A_1} \Delta F(x_1,x_2)\,dx_1\right]dx_2$$
$$+ \int_{A_1} U_{11}(x_1,a_2)\left[\int_0^{x_1} \Delta F(s,a_2)\,ds\right]dx_1$$
$$- \int_{A_1}\int_{A_2} U_{112}(x_1,x_2)\left[\int_{A_1} \Delta F(s,x_2)\,ds\right]dx_1\,dx_2.$$

Finally, integrating the first term in the RHS term above by parts and evaluating the other terms yields

$$\Delta W_U = -U_2(a_1,a_2)\Delta H_2(a_2) + \int_{A_2} U_{22}(a_1,x_2)\Delta H_2(x_2)\,dx_2 - U_1(a_1,a_2)\Delta H_1(a_1) + \int_{A_2} U_{12}(a_1,x_2)\Delta H_1(a_1;x_2)\,dx_2 + \int_{A_1} U_{11}(x_1,a_2)\Delta H_1(x_1)\,dx_1 - \int_{A_1}\int_{A_2} U_{112}(x_1,x_2)\Delta H_1(x_1;x_2)\,dx_1\,dx_2.$$

Since Condition (C) implies $\Delta H_1(x_1) \leqslant 0$, $\forall x_1 \in A_1$, the conclusion follows from examining the signs of each term.

A.2. *Proof of Proposition 2*

A.2.1. *Proof of (L_1)*

We consider two cases. In both cases, we choose a fixed value of x_2 in A_2.

Case 1. Assume that $x_2 \in A_2$ is such that $F_2^*(x_2) \leqslant F_2(x_2)$. We prove that $\Delta H_1(x_1;x_2) \leqslant 0$, $\forall x_1 \in A_1 \Rightarrow \mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F^*_{x_2}}(p)$, $\forall p \in [0, F_2^*(x_2)]$.

We use Young's inequality (see Genet [22, Theorem 1, p. 195]), which reduces in this case to an equality since we apply it to the corresponding bounds. Starting with the definition of $H_1(x_1;x_2)$, we obtain

$$\int_0^{x_1} F_{x_2}(s)\,ds = x_1 F_{x_2}(x_1) - \int_0^{F_{x_2}(x_1)} F_{x_2}^{-1}(t)\,dt,$$

or with $q = F_{x_2}(x_1) \in [0, F_2(x_2)]$:

$$H_1(x_1;x_2) = F_{x_2}^{-1}(q)q - \int_0^q F_{x_2}^{-1}(t)\,dt.$$

Using a similar expression for $H_1^*(x_1;x_2)$ with $q^* = F^*_{x_2}(x_1) \in [0, F_2^*(x_2)]$, we have

$$\Delta H_1(x_1;x_2) = qF_{x_2}^{-1}(q) - q^*F_{x_2}^{*-1}\big(q^*\big) - \left[\int_0^q F_{x_2}^{-1}(t)\,dt - \int_0^{q^*} F_{x_2}^{*-1}(t)\,dt\right]. \tag{10}$$

Since $F_2^*(x_2) \leqslant F_2(x_2)$, we have $F^*_{x_2}(x_1) = F^*(x_1,x_2) \leqslant F^*(a_1,x_2) = F_2^*(x_2) \leqslant F_2(x_2)$.

We consider three sub-cases, according to whether $q \lesseqgtr q^*$. If $q > q^*$, then

$$\Delta H_1(x_1;x_2) = qF_{x_2}^{-1}(q) - q^*F_{x_2}^{*-1}\big(q^*\big) - \int_{q^*}^{q} F_{x_2}^{-1}(t)\,dt - \big[\mathcal{C}_{F_{x_2}}\big(q^*\big) - \mathcal{C}_{F^*_{x_2}}\big(q^*\big)\big].$$

Using $F_{x_2}^{-1}(q) = F_{x_2}^{*-1}(q^*) = x_1$,

$$\Delta H_1(x_1; x_2) = \left[F_{x_2}^{-1}(q)\big(q - q^*\big) - \int_{q^*}^{q} F_{x_2}^{-1}(t)\,dt \right] - \big[\mathcal{C}_{F_{x_2}}\big(q^*\big) - \mathcal{C}_{F_{x_2}^*}\big(q^*\big)\big].$$

Applying the mean-value theorem for integrals, the term in brackets is always strictly positive if $q > q^*$ because F_{x_2} is continuous and strictly increasing. Therefore, $\Delta H_1(x_1; x_2) \leqslant 0 \Rightarrow \mathcal{C}_{F_{x_2}}(F_{x_2}^*(x_1)) - \mathcal{C}_{F_{x_2}^*}(F_{x_2}^*(x_1)) > 0$. Since $F_{x_2}^*(x_1) \in [0, F_2^*(x_2)]$, $\Delta H_1(x_1; x_2) \leqslant 0$, $\forall x_1 \in A_1$ implies $\mathcal{C}_{F_{x_2}}(p) > \mathcal{C}_{F_{x_2}^*}(p)$, $\forall p \in [0, F_2^*(x_2)]$. The proof is similar for $q^* > q$. Finally, if $q = q^*$, the terms in brackets vanish and $\Delta H_1(x_1; x_2) \leqslant 0$ implies $\mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F_{x_2}^*}(p)$. This situation corresponds to the intersection points x_1 of the projected distribution functions. The impossibility of considering quantiles up to 1 prevents us from proposing a straightforward reciprocal result as opposed to what is possible in the unidimensional case.

Case 2. Assume $x_2 \in A_2$ is such that $F_2(x_2) < F_2^*(x_2)$. We prove that $\Delta H_1(x_1; x_2) \leqslant 0$, $\forall x_1 \in A_1 \Rightarrow \mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F_{x_2}^*}(p)$, $\forall p \in [0, F_2(x_2)]$.

We start again with (10). By assumption, $\Delta H_1(x_1; x_2) \leqslant 0$ for all x_1. In particular, this inequality holds for all $x_1 \in A_1$ such that $F_{x_2}^*(x_1) \leqslant F_2(x_2)$. In this case, the last part of the necessity of the proof of Case 1 from (10) remains the same. Therefore, we can deduce that $\Delta H_1(x_1; x_2) \leqslant 0 \Rightarrow \mathcal{C}_{F_{x_2}}(F_{x_2}^*(x_1)) - \mathcal{C}_{F_{x_2}^*}(F_{x_2}^*(x_1)) \geqslant 0$. Since $F_{x_2}^*(x_1) \in [0, F_2(x_2)]$, $\Delta H_1(x_1; x_2) \leqslant 0$, $\forall x_1 \in A_1$ implies $\mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F_{x_2}^*}(p)$, $\forall p \in [0, F_2(x_2)]$. Statement ($L_1$) follows with $\mathcal{C}_{F_{x_2}}(p) > \mathcal{C}_{F_{x_2}^*}(p)$ at all points x_1 for which the corresponding projected distribution functions do not intersect.

A.2.2. Proof of (L_2)

Assume $x_2 \in A_2$ is such that $F_2(x_2) \leqslant F_2^*(x_2)$.

In view of (L_1), it suffices to prove that $\mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F_{x_2}^*}(p)$, $\forall p \in [0, F_2(x_2)]$ implies $\Delta H_1(x_1; x_2) \leqslant 0$, $\forall x_1 \in A_1$.

Suppose that there exists $q \in [0, F_2(x_2)]$ such that $\mathcal{C}_{F_{x_2}}(q) \geqslant \mathcal{C}_{F_{x_2}^*}(q)$. Then, there exists $x_1 \in A_1$ such that $x_1 = F_{x_2}^{-1}(q)$ and there exists $q^* = F_{x_2}^*(x_1)$. Starting from Eq. (10) (which remains valid) and using $F_2(x_2) \leqslant F_2^*(x_2)$, which implies that $q = F(x_1, x_2) \leqslant F(a_1, x_2) \leqslant F_2^*(x_2)$, one gets in the case $q^* > q$:

$$\Delta H_1(x_1; x_2) = q F_{x_2}^{-1}(q) - q^* F_{x_2}^{*-1}\big(q^*\big) + \int_{q}^{q^*} F_{x_2}^{*-1}(t)\,dt$$
$$- \left[\int_0^q F_{x_2}^{-1}(t)\,dt - \int_0^q F_{x_2}^{*-1}(t)\,dt \right],$$

or using $F_{x_2}^{-1}(q) = F_{x_2}^{*-1}(q^*)$:

$$\Delta H_1(x_1; x_2) = \left[\int_q^{q^*} F_{x_2}^{*-1}(t)\,dt - F_{x_2}^{*-1}\big(q^*\big)\big(q^* - q\big) \right] - \big[\mathcal{C}_{F_{x_2}}(q) - \mathcal{C}_{F_{x_2}^*}(q)\big].$$

Applying the mean-value theorem for integrals, the first term in brackets is strictly negative since $F^*_{x_2}$ is continuous and strictly increasing. Then, $\mathcal{C}_{F_{x_2}}(q) - \mathcal{C}_{F^*_{x_2}}(q) \geqslant 0$ implies $\Delta H_1(F^{-1}_{x_2}(q); x_2) < 0$. Since the range of $F^{-1}_{x_2}(p)$ is A_1 for $p \in [0, F_2(x_2)]$, $\mathcal{C}_{F_{x_2}}(p) \geqslant \mathcal{C}_{F^*_{x_2}}(p)$, $\forall p \in [0, F_2(x_2)]$ implies $\Delta H_1(x_1; x_2) < 0$, $\forall x_1 \in A_1$. The derivation is similar for the case $q < q^*$. In the case where $q = q^*$, the first term in brackets vanishes and $\mathcal{C}_{F_{x_2}}(p) - \mathcal{C}_{F^*_{x_2}}(p) \geqslant 0$, for all $p \in [0, F_2(x_2)]$ implies $\Delta H_1(x_1; x_2) \leqslant 0$.

A.3. Proof of Proposition 3

Starting from the final expression for ΔW_U in the proof of Proposition 1 and integrating with respect to x_2 the second term on the RHS term by parts, with $U_{22}(a_1, x_2)$ continuous in x_2, we obtain

$$\begin{aligned}\Delta W_U = {} & -U_2(a_1, a_2)\Delta H_2(a_2) + U_{22}(a_1, a_2)\Delta L_2(a_2) - \int_{A_2} U_{222}(a_1, x_2)\Delta L_2(x_2)\, dx_2 \\ & - U_1(a_1, a_2)\Delta H_1(a_1) + \int_{A_2} U_{12}(a_1, x_2)\Delta H_1(a_1; x_2)\, dx_2 \\ & + \int_{A_1} U_{11}(x_1, a_2)\Delta H_1(x_1)\, dx_1 - \int_{A_1}\int_{A_2} U_{112}(x_1, x_2)\Delta H_1(x_1; x_2)\, dx_1\, dx_2.\end{aligned}$$

The conclusion follows.

References

[1] S.C. Athey, Characterizing properties of stochastic objective functions, MIT Working Paper 96-1R, revised, 2000.
[2] A.B. Atkinson, On the measurement of inequality, J. Econ. Theory 2 (1970) 244–263.
[3] A.B. Atkinson, F. Bourguignon, The comparison of multi-dimensioned distributions of economic status, Rev. Econ. Stud. 49 (1982) 181–201.
[4] A.B. Atkinson, F. Bourguignon, Income distribution and differences in needs, in: G.R. Feiwel (Ed.), Arrow and the Foundation of the Theory of Economic Policy, Macmillan, London, 1987, pp. 350–370.
[5] A.B. Atkinson, F. Bourguignon, Income distribution and economics, in: A.B. Atkinson, F. Bourguignon (Eds.), Handbook of Income Distribution, North-Holland, 2000, pp. 1–85.
[6] J. Banks, P. Johnson, Equivalence scale relativities revisited, Econ. J. 104 (1994) 883–890.
[7] S. Bazen, P. Moyes, International comparisons of income distributions, Res. Econ. Inequal. 9 (2003) 85–104.
[8] P. Billingsley, Probability and Measure, Wiley–Interscience, New York, 1986.
[9] F. Bourguignon, Family size and social utility: Income distribution dominance criteria, J. Econometrics 42 (1989) 67–80.
[10] C. Chambaz, E. Maurin, Atkinson and Bourguignon's dominance criteria: Extended and applied to the measurement of poverty in France, Rev. Income Wealth 44 (1998) 497–513.
[11] M. Denuit, L. Eeckhoudt, I. Tsetlin, R.L. Walker, Multivariate concave and convex stochastic dominance, Working Paper 2010/29/AS, INSEAD.
[12] M. Denuit, C. Lefèvre, M. Mesfioui, A class of bivariate stochastic orderings, with applications in actuarial sciences, Ins.: Mathematics Econ. 29 (1999) 31–50.
[13] M. Denuit, M. Mesfioui, Generalized increasing convex and directionally convex orders, J. Appl. Probab. 47 (2010) 264–276.
[14] R. Dworkin, What is equality? Part 2: Equality of resources, Philos. Publ. Affairs 10 (1981) 283–345.
[15] U. Ebert, Using equivalence income of equivalent adults to rank income distributions, Soc. Choice Welfare 16 (1999) 233–258.
[16] L. Eeckhoudt, B. Rey, H. Schlesinger, A good sign for multivariate risk taking, Manag. Sci. 53 (2007) 117–124.

[17] P.C. Fishburn, I.H. Lavalle, Stochastic dominance on unidimensional grids, Math. Oper. Res. 20 (1995) 513–525.
[18] P.C. Fishburn, R.G. Vickson, Theoretical foundations of stochastic dominance, in: G.A. Whitmore, M.C. Findlay (Eds.), Stochastic Dominance: An Approach to Decision Making under Risk, Heath, Lexington, MA, 1978, pp. 37–113.
[19] M. Fleurbaey, Fairness, Responsibility and Welfare, Oxford Univ. Press, Oxford, 2008.
[20] M. Fleurbaey, C. Hagneré, A. Trannoy, Welfare analysis when equivalence scales differ, J. Econ. Theory 110 (2003) 309–336.
[21] J.E. Foster, A.F. Shorrocks, Poverty orderings and welfare dominance, Soc. Choice Welfare 5 (1988) 179–198.
[22] J. Genet, Mesure et Intégration, Vuibert, Paris, 1976.
[23] N. Gravel, P. Moyes, Ethically robust comparisons of bidimensional distributions with an ordinal attribute, J. Econ. Theory 147 (4) (2012) 1384–1426, in this issue.
[24] S.P. Jenkins, P.J. Lambert, Ranking income distributions when needs differ, Rev. Income Wealth 39 (1993) 337–356.
[25] S.C. Kolm, The optimal production of social justice, in: H. Guitton, J. Margolis (Eds.), Public Economics, Macmillan, London, 1969, pp. 145–200.
[26] S.C. Kolm, Multidimensional egalitarianisms, Quart. J. Econ. 91 (1977) 1–13.
[27] P.J. Lambert, X. Ramos, Welfare comparisons: Sequential procedures for homogeneous populations, Economica 69 (2002) 549–562.
[28] P. Moyes, Comparisons of heterogeneous distributions and dominance criteria, Econ. Prévision 138–139 (1999) 125–146 (in French).
[29] A. Müller, Stochastic orders generated by integrals: A unified study, Adv. in Appl. Probab. 29 (1997) 414–428.
[30] C. Muller, A. Trannoy, A Dominance approach to the appraisal of the distribution of well-being across countries, J. Public Econ. 95 (2011) 239–246.
[31] J. Roemer, Theories of Distributive Justice, Harvard Univ. Press, Cambridge, MA, 1996.
[32] E. Schokkaert, K. Devooght, Responsibility-sensitive fair compensation in different cultures, Soc. Choice Welfare 21 (2003) 207–242.
[33] A.K. Sen, The Standard of Living, Cambridge Univ. Press, Cambridge, 1987.
[34] A.F. Shorrocks, Ranking income distributions, Economica 50 (1983) 1–17.
[35] A.F. Shorrocks, Inequality and welfare evaluation of heterogeneous income distributions, J. Econ. Inequal. 2 (2004) 193–218.
[36] A.F. Shorrocks, J.E. Foster, Transfer sensitive inequality measures, Rev. Econ. Stud. 54 (1987) 485–497.
[37] A. Trannoy, Multidimensional egalitarianism and the dominance approach: a lost paradise, in: F. Farina, E. Savaglio (Eds.), Inequality and Economic Integration, Routledge, London, 2006, pp. 282–302.