Journal of Public Economics 95 (2011) 239–246

Contents lists available at ScienceDirect

Journal of Public Economics

journal homepage: www.elsevier.com/locate/jpube

A dominance approach to the appraisal of the distribution of well-being across countries☆

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ARTICLE INFO

Article history:
Received 4 January 2010
Received in revised form 30 September 2010
Accepted 25 October 2010
Available online 13 November 2010

JEL classification:
O15
D31
I9

Keywords:
Multidimensioned welfare
Stochastic dominance
Human Development Index

ABSTRACT

This paper proposes a dominance approach to study inequality of well-being across countries. We consider a class of well-being indices based on the three attributes used in the HDI (Human Development Index). Indices are required to satisfy: preference for egalitarian marginal distributions of income, health and education, ALEP substitution of attributes and priority to poor countries in allocating funds to enhance health and education. We exhibit sufficient conditions for checking dominance over the defined class of well-being indices. We apply our method to country data from 2000 to 2005. The deterioration in health conditions in poor countries is why welfare improvements at the world level cannot be ascertained.

1. Introduction

Over the course of the economic literature, the attention of analysts comparing the well-being of countries has shifted from an assessment based solely on per capita GDP to the examination of several attributes. Namely, life expectancy, literacy rates, mortality and morbidity statistics and other socioeconomic indicators have often been considered in addition to income per capita. Armatya Sen played a key role in this process, opening a broader perspective to human development by introducing the notions of functioning and capabilities (Sen, 1992, 2009). It has been repeatedly observed that education and health should be among the major ingredients of some fundamental functionings.

However, appraising the inequality of the distribution of well-being across countries raises difficulties in a multi-dimensional setting. In a casual approach, the information provided by the attributes describing a country situation is aggregated in some indicator. For instance, one popular measure of well-being is the HDI (Human Development Index) published every year by UNDP in the Human Development Report since 1990. Sen laid the intellectual foundations of this index (Anand and Sen, 1993, 1999), which is based on three attributes: life expectancy at birth, education as measured by a weighed mean of the adult literacy rate and of the combined primary, secondary and tertiary gross enrollment ratio, and real GDP per capita at purchasing power parity. The HDI[1] is obtained by: (1) placing each country on a scale of 0 to 100 with respect to each attribute, and (2) by computing a simple arithmetic mean of the attributes.

Yet, there is no uncontroversial way of aggregating carrots and potatoes. It should be more sensible to straightforwardly face the intrinsic multi-dimensional feature of the measurement problem at hand and to agree on some qualitative properties that a "well-behaved index" should satisfy. In doing so, on the one hand, we leave some room for disagreement, and, on the other hand, we can deal with a whole class of well-being indices rather than with a unique and somewhat arbitrary index. This kind of problem is common in inequality analysis.

The approach to social well-being by using stochastic dominance responds to such concern. It was pioneered by Kolm (1969), Atkinson

☆ This research was funded by a grant from British Academy. We are grateful to Tony Atkinson for his comments, Laure Schultz for her excellent research assistance and to Pierre-Henri Bono for his help with the graphics.

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[1] Each of these components is normalized by a simple procedure. For instance, an estimate x for the GDP component, means that the log GDP of the country is at x% between the two reference values, which are the minimum and the maximum of log GDPs for the countries registered at a given period. Such normalization, proposed by Anand and Sen (1999), corrects deficiencies associated with a previous normalization.

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doi:10.1016/j.jpubeco.2010.10.012

(1970) and Sen (1973). It relies on proposing statistical comparison criteria for distributions, which are derived by taking into account the indeterminacy of some aggregating indicator—e.g., an utility function in the case of individuals and a well-being index in the case of countries.

Dominance analysis consists in seeking unanimity among classes of social welfare functions over the ranking of pairs of allocations. What we propose is a multi-dimensional dominance analysis applied to well-being indicators of the type of the HDI. The social welfare function, here expressed at the world level, is assumed additively separable with respect to well-being indicators defined at the country level. Of course, in doing so we accept to be sometimes unable to attain a definite conclusion for the comparison since the obtained criteria are incomplete. Nevertheless, in the situations where we can reach a conclusion, the results are robust and cannot be easily dismissed.

The literature on multi-dimensioned welfare analysis can be traced back to Kolm's (1977) paper, where every attribute is considered symmetrically. There has been a bunch of theoretical papers devoted to this topic (see e.g. Marshall and Olkin, 1979, chapter 15; Atkinson and Bourguignon, 1982; Le Breton, 1986; Koshevoy, 1995, 1998; Koshevoy and Mosler, 1996; Moyes, 1999; Bazen and Moyes, 2003), with some empirical applications (Duclos et al., 2006a,b). In particular, Atkinson and Bourguignon (1982) propose dominance relationships for various classes of utilities defined by their derivatives up to the fourth order. Nevertheless, it seems fair to say that no simple criterion for checking multi-dimensional stochastic dominance has reached broad support among applied economists and even among theorists. Our criterion can be viewed as intermediate between the weaker criterion of Atkinson and Bourguignon (1982) and their stronger criterion. Checking multi-dimensional dominance can be seen as a somewhat preliminary task before engaging in the estimation of specific multidimensional inequality indices as done by Decancq et al. (2009).

In this article, we consider a three-variable distribution problem. This approach is rather uncommon since the main bulk of the literature has limited its attention to the case of a bivariate distribution (see Gravel and Mukhopadhyay, 2009, for an extension of Atkinson and Bourguignon criteria to more than two goods). Section 2 presents the considered class of well-being indicators. Section 3 reports the result. Section 4 provides an empirical application to the distribution of welfare among countries between 2000 and 2005. Finally, Section 4 concludes.

2. A class of well-being indicators

Our starting point is to accept that well-being at a country level be based on the three attributes considered in the HDI, namely: per capita GDP, life expectancy at birth and educational attainment. These variables are supposed to be continuous.[2] We denote the joint cumulative distribution function (cdf) of these three indicators across countries as $F(x_1, x_2, x_3)$, where x_1 is per capita GDP, x_2 is life expectancy at birth, and x_3 is educational attainment.

It seems that there is an equal interest for considering population-weighed distributions and unweighed distributions of attributes across countries (see for instance Milanovic, 2003, for income inequality). In the first case, China and India matter a lot for the evolution of world welfare, while in the second case each country counts for an observation in the social welfare function, irrespective of its demographic weight. In the weighed case, F is the cdf of the attributes at individual level, assuming that each individual in a given country is assigned the vector of attributes of his/her country. In the unweighed case, F is simply the cdf at country level. To simplify the proofs' exposition, we assume that F admits a density f defined on a finite support $X_1 \times X_2 \times X_3$ with $X_1 = [0, a_1]$, $X_2 = [0, a_2]$ and $X_3 = [0, a_3]$. Using the same notations as Atkinson and Bourguignon (1982), we define the world social welfare function as

$$W_I := \int_0^{a_1} \int_0^{a_2} \int_0^{a_3} I(x_1, x_2, x_3) f(x_1, x_2, x_3) dx_1 dx_2 dx_3,$$

where I is the well-being function at country level, assumed to be continuously differentiable to the required degree. The partial derivatives with respect to each variable are denoted by subscripts. The change in welfare between two probability distributions f and f^* is

$$\Delta W_I := \int_0^{a_1} \int_0^{a_2} \int_0^{a_3} I(x_1, x_2, x_3) \Delta f(x_1, x_2, x_3) dx_1 dx_2 dx_3 \quad (1)$$

where Δf denotes $f - f^*$.

Dominance is defined as unanimity for a family of social welfare functions based on a specific set of well-being functions.

Definition 1. f dominates f^* for a family $\mathcal{I}$ of well-being functions if and only if $\Delta W_I \geq 0$ for all well-being functions I in $\mathcal{I}$. This is denoted $f D_{\mathcal{I}} f^*$.

The considered class of well-being functions is the following[3]:

$$\mathcal{I} = \left\{ I : X_1 \times X_2 \times X_3 \to \mathbb{R} \text{ such that } \begin{array}{c} I_1, I_2, I_3 \geq 0; \\ I_{11} \leq 0, I_{22} \leq 0, I_{33} \leq 0; \\ I_{12} \leq 0, I_{13} \leq 0, \ I_{23} = 0; \\ I_{121} \geq 0, I_{131} \geq 0 \end{array} \right\}$$

With this class, the marginal well-being gain induced by an increase in any attribute is supposed to be identical across countries at given levels of attributes. This hypothesis captures a requirement of anonymity at the country level in the unweighed interpretation and at the individual level in the weighed interpretation.

International aid is sometimes understood as equivalent to increasing per capita GDP since we focus on monetary transfers. However, aid is also sometimes donated with specific assignations, for example with the purpose of increasing life expectancy or education. This is accounted for by the properties of increasing monotonicity of the welfare utility with respect to each dimension. Marginal well-being gains are assumed to be not only positive but also decreasing in all dimensions. This condition implies that each attribute is good for welfare and there is a social preference at the world level for more egalitarian marginal distributions. In one-dimensional settings, dominance for concave and increasing utility functions is known to be equivalent to the Generalized Lorenz test introduced by Shorrocks (1983). Therefore, a not less egalitarian distribution, according to the Generalized Lorenz test in each dimension, appears to be a prerequisite for accepting that welfare has improved at the world level.

Indeed, the negative signs of the direct second derivatives ($I_{11} \leq 0$, $I_{22} \leq 0$, $I_{33} \leq 0$) can each be related to the pervasive Pigou–Dalton axiom in the one-dimensional welfare literature. As a matter of fact, whenever the first argument is income, the condition $I_{11} \leq 0$, corresponds to inequality reduction by applying Pigou–Dalton transfers among individuals, that is: inequality aversion in income.

[2] These three variables are supposed to have a cardinal meaning. That is, they are defined up to any linear transformation.

[3] Since I is assumed to be twice continuously differentiable, from Young's theorem, we necessarily have: $I_{21} = I_{12} \leq 0$, $I_{31} = I_{13} \leq 0$ and $I_{32} = I_{23} = 0$.

However, in the cases and for the attributes that do not correspond to income (or to a simple transformation of income into per capita consumption, for example), the justification of these signs may be less obvious. For instance, controlling transfers of health or education quantities does not seem practically implementable. Nonetheless, Pigou–Dalton type transfer axioms have been extensively used for non-income dimensions, for example by Tsui (1999) and other authors. This suggests that the corresponding intuition of inequality aversion in the considered attribute does not require the practical possibility of transfers, while only its abstract consideration.

If the distributions of each attribute were independent, Generalized Lorenz tests in each dimension would be sufficient to yield a reasonable criterion. But we empirically know that our three attributes of interest are positively correlated. Hence, some assumptions are needed to convey the intuition that a reduction in the statistical links of these variables would improve welfare.

In the case of per capita GDP and life expectancy at birth, we capture the latter idea by imposing a negative sign on the second partial cross-derivative with respect to these two attributes. The same is required for the second partial cross-derivative of the well-being function with respect to per capita GDP and education attainment. Strictly speaking, these assumptions indicate that the marginal increase in well-being associated with a rise in income per capita is decreasing with the level of the other two variables.

We provide an intuitively appealing reinterpretation of the cross-partial derivatives in terms of a 'priority to international aid'. They imply that, when comparing two equally poor countries (with the same per capita GDP) competing for international aid, priority should be given to the country with the lower health or the lower educative performance. A consequence of this is that monetary transfers from some countries to poorer countries with worse health or worse education standards are always good for welfare. After such transfers, the income distribution across countries will be less correlated to the distributions of the two other attributes. Another justification of this assumption is that it is equivalent to assuming that income per capita and health (or education) are ALEP substitutes in well-being. Thus, income can be used to compensate for deficiencies in health and education.

Our interpretation of conditions on cross-derivatives in terms of priority to international aid allows us to give a precise and intuitive meaning to the properties of the index, with practical consequences for the empirical test. Moreover, it leads us to questioning explicitly the standard assumptions of similar signs for all possible combinations of cross-partial derivatives. Notably, even if we specify that the cross-partial derivation between income and health, and between income and education, is non-positive, we argue that there is not enough motivation to impose the same conditions between education and health. As a matter of fact, aid policies directed respectively to education programs and health interventions are generally separated, and nationally monitored and managed by different national administrations or ministries. Even at international level, different international organizations (respectively UNICEF and WHO), or different departments within organizations as in the World Bank, separately deal with these domains of social issues. In these conditions, there is little reason to practically consider possible cross-compensations between health and education domains.

Cancelling I_{23} implies that the aggregate well-being function is additively separable with respect to life expectancy at birth and educational attainment. Indeed for the purpose of welfare analysis, it often seems sensible for practical policies, at least in first approximation, to consider that the marginal gain in welfare from one additional year of life expectancy at birth is not substantially affected by education levels. In other words, even if it can be purported in policy circles that income can used to compensate for bad health or bad education, it is less practically and politically attractive to impose that a good health could be used as a substitute for a bad education, or the opposite. Of course, it can be argued that the better educated people are, the more able they are to enjoy life. However, introducing such an assumption would imply some undesirable conclusion: an international aid program transferring resources to improve health conditions would have to be targeted to countries with the highest educational level!

Note that the HDI aggregate welfare formula already satisfies this additive separability assumption. Under these conditions, the aggregate welfare index can be written as follows, with φ and ψ both three time differentiable

$$I(x_1, x_2, x_3) = \varphi(x_1, x_2) + \psi(x_1, x_3) \quad (2)$$

The negative sign of the cross-partial derivatives (here, $I_{12} \leq 0$ and $I_{13} \leq 0$) is an assumption already present in Atkinson and Bourguignon (1982), who justify it by the example of an utility function $U(x_1, x_2) = V[\Phi(x_1, x_2) + \Psi(x_1, x_3)]$ with $\Phi', \Psi' > 0$, $\Phi'', \Psi'' \leq 0$, $V' > 0$, $V'' \leq 0$. They also emphasize the 'natural starting point' of an additive welfare function (in a sense, corresponding to our assumption $I_{23} = 0$). Nonetheless, there might also be situations where a non-zero partial derivative between health and education is sensible. For example, that could be the case if one wanted to account for crossed effects of education and health in home care technologies. However, it might be more reasonable to explicitly account for these new constraints in the theoretical setting. Meanwhile, generating terms with this derivative in the social welfare expansion by using integration by parts has also the consequence of introducing terms with fourth-degree partial derivatives. For example, we would have to assume $I_{1123} \leq 0$ to obtain exploitable results. However, this latter condition seems harder to justify normatively and looks rather arbitrary.

Atkinson and Bourguignon provide some intuitive feeling and motivation for the sign of U_{12} by invoking transformations of the joint bivariate density of two attributes such that the marginal distributions are unchanged, but the correlation between attributes is reduced. It is this type of transformation that makes multidimensional welfare analysis intrinsically different from the one-dimensional case. Assumptions on cross-partial derivatives of the welfare criterion allow analysts to compare distributions with identical margins but distinct degrees of correlation.

Likewise, the sign of the cross-partial derivatives of $I(x_1, x_2, x_3)$ is underlying the 'correlation increasing majorization (CIM)' in Tsui (1999). A matrix X of attributes by individuals dominates another matrix Y in the sense of CIM whenever X may be derived from Y by a permutation of columns and a finite sequence of correlation increasing transfers at least one of which is strict. However, Tsui's approach implies that increasing correlation leads to reduced welfare, while our hypotheses are not sufficient to prove that it is always the case. Consider a simple example. Let be two individuals $i = 1, 2$, with respective incomes $y_1 > y_2$ and respective education levels $e_1 < e_2$. The situation where the individuals would have exchanged their education levels corresponds to an increasing correlation. Would this yield a reduction of welfare with utilities in our class? A first-order Taylor expansion of the welfare gives

$$dW = de \cdot (U_e(y_1, e_1) - U_e(y_2, e_2)).$$

To obtain a negative number with $U_e(y_1, e_1) - U_e(y_2, e_2) = [U_e(y_1, e_1) - U_e(y_2, e_1)] + [U_e(y_2, e_1) - U_e(y_2, e_2)]$ is not ensured as the first bracket is negative (because of $U_{12} \leq 0$) and the second one is positive because of $U_{22} \leq 0$.

The role of correlation increasing majorization is emphasized in Bourguignon and Chakravarty (2003), as defining whether the dimensions are considered to be complement or substitutes in the

ALEP sense. They note that, for example, if income and education are considered as substitutes (i.e. with our assumptions: $U_{13}<0$), then the drop in poverty due to a unit increase in income is smaller for people who have substantial education level than for people with low education. In general, under subgroup decomposition, which is our case, increasing correlation for substitute attributes induces a welfare drop.[4]

Even under such separability structure as in Eq. (2), it is unlikely that one generates any dominance criteria with substantial discriminatory power without adding further restrictions. This justifies introducing the last two assumptions of positivity of third-degree partial cross-derivatives. These assumptions are best understood if we note that minus the value of I_{12} (respectively I_{13}) can be interpreted as an index of priority to international aid for compensating low life expectancy (respectively education attainment) with income transfers. Then, requiring that minus I_{121} (respectively minus I_{131}) is negative implies that the above priority index decreases with the country income per capita. In other words, the countries with the highest claim to international aid for compensating health conditions are the poorest ones, a somewhat reasonable request. In contrast, it is well-known that despite a very high share of GDP devoted to health expenses, the US is far from being the leader in life expectancy among OECD countries. Yet, improving health conditions in this country through international aid would not seem to command a widespread support, consistently with our hypotheses.

For comparison the HDI formula is as follows. The HDI is based on three indicators: GDP per capita measured in PPP US\$ (x_1), life expectancy at birth (x_2) and educational attainment (x_3). Education attainment is defined as 2/3 times the adult literacy rate + 1/3 times the combined gross primary, secondary and tertiary enrolment ratio. Then, each of these indicators used for the HDI is normalized by comparing its value to benchmark minimum and maximum values. These boundary values are: 25 years and 85 years for life expectancy at birth, 0% and 100% for the adult literacy rate, 0% and 100% for the gross enrolment ratio, \$100 and \$40,000 (PPP US\$) for GDP per capita. Alternatively, taking lower and upper bounds over the studied sample has been proposed.

For health and education, the normalized variable $\hat{x}_i$ used in the index is defined as

$$W^i(x_i) = \frac{x_i - x_i^m}{x_i^M - x_i^m}, \quad i = 2, 3.$$

where x_i^m is the minimum benchmark and x_i^M is the maximum benchmark for indicators $i = 1, 2, 3$. The normalized variable for per capita GDP is

$$W^1(x_1) = \frac{\log x_1 - \log x_1^m}{\log x_1^M - \log x_1^m}.$$

where the logarithm reflects the fact that achieving a respectable level of human development does not require exaggerated income level.

Finally, the HDI is the simple average of the three normalized indicators for GDP per capita, life expectancy and educational attainment:

$$I(x_1, x_2, x_3) = \frac{W^1(x_1) + W^2(x_2) + W^3(x_3)}{3}.$$

[4] It is also interesting to note, e.g., in Tsui (1999), that CIM is independent of various Pigou–Dalton majorizations. That is: the conditions for cross-partials on the one hand and direct second-order partials on the other hand can be specified independently. This supports our approach of separately imposing normatively conditions corresponding to these respective derivatives.

Clearly, this formula of the HDI shows that all cross-partial derivatives of interest, and third-degree derivatives, are equal to zero. This is due to the separability between income and the two other arguments. If we could accept such separability between health and education in the index as reflecting common practices in administering international aid, the separability of income is much more debatable since it excludes practically relevant policies of monetary transfers to destitute households in health or education characteristics. The HDI belongs to our proposed family, even if as an extreme point, but it does not allow for relevant policy trade-offs between income and health (or education). Moreover, the HDI does not account for inequality aversion in the health and education dimensions since it is only concave in the income dimension.

We can propose another formula for an alternative to the HDI, which strictly respects the signs required for belonging to the class $\mathcal{I}$. For example, this expression

$$I(W_1, W_2, W_3) = 1 - \left(\frac{1}{f(W_1) + f(W_2)} + \frac{1}{g(W_1) + g(W_3)} \right)$$

has the required properties provided that f and g are strictly positive, increasing and concave functions. For an index between 0 and 1, $f(x) = g(x) = \sqrt{x}$ is a simple choice. In this expression, we admitted that the normalization of the three variables, income, health and education is suitable.

3. The result

In performing the integration of Eq. (1), it is convenient to define the marginal distribution $F_1(x_1) = \int_0^{a_3} \int_0^{a_2} \int_0^{x_1} f(s, r, t) dsdrdt$ and the corresponding expressions for $F_2(x_2)$ and $F_3(x_3)$. Furthermore, we define some second-degree stochastic dominance terms:

$$H_i(x_i) := \int_0^{x_i} F_i(s) ds, \quad i = 1, 2, 3$$

$$H_i\left(x_i; x_j, x_k\right) := \int_0^{x_i} F\left(r, x_j, x_k\right) dr$$

and

$$H_i\left(x_i; x_j\right) := H_i\left(x_i; x_j, a_k\right) \text{for any } i, j, k.$$

The following proposition gives sufficient conditions to check dominance according to class $\mathcal{I}$.

Proposition 1. *Let f and f* be densities. If all of the following conditions hold*

$$\Delta H_i(x_i) \le 0, \forall x_i \in X_i \quad i = 2, 3 \qquad (A)$$
$$\Delta H_1(x_1; x_2) \le 0, \forall x_i \in X_i, \quad i = 1, 2 \qquad (B)$$
$$\Delta H_1(x_1; x_3) \le 0, \forall x_i \in X_i, \quad i = 1, 3. \qquad (C)$$

then

$f D_{\mathcal{I}} f^*$

The three families of conditions which guarantee the existence of a dominance relation are easy to be implemented. Checking condition (A) is equivalent to check dominance according to the Generalized Lorenz test on the marginal distributions of life expectancy at birth and educational attainment. Moreover, one has to verify conditions (B) and (C) which are symmetric in terms of the roles played by x_2 and x_3. One first considers the value of the joint cdf of health and education at the upper bound for the health variable (respectively educational variable) in the former (respectively latter) condition. Then, one performs a single integration with respect to the income per capita variable. Finally, one checks whether the value of the integral is smaller for the dominant distribution than for the dominated distribution, for any couple of income per capita and life expectancy (respectively educational attainment). Note that conditions (B) and (C) imply that the Generalized Lorenz test is also satisfied for the marginal distribution of per capita GDP.

Foster and Shorrocks (1988) showed that second-degree stochastic dominance conditions are equivalent to poverty gap conditions. Conditions (B) and (C) can also be rephrased in terms of poverty gaps. Condition (B) is equivalent to require that the absolute income poverty gap decreases for the 'health-destitute' countries, for all income poverty lines, and this whatever the considered threshold of minimal health used to define 'health-destitution'. Similarly, condition (C) is equivalent to require that the absolute income poverty gap decreases for the 'education-destitute' countries, for all income poverty lines and this whatever the considered education level.

Condition (A) is clearly necessary. The corresponding proof replicates standard arguments (see Fishburn and Vickson, 1978). Showing that conditions (B) and (C) are also necessary would be harder since we have to isolate a function belonging to the class of interest such that if either condition (B) or (C) is violated, then the welfare change goes in the wrong direction.

It remains to mention a potential difficulty raised by the normalization implemented when computing the HDI. One may ask whether checking dominance conditions (A), (B) and (C) on the non-transformed variables x_i is sufficient to have the same three conditions satisfied on the normalized variables $W^i(x_i)$. It is easily established that the answer is positive when the normalized variables are concave and increasing transformations of the original ones, by using theorem A2. p. 116 in Marshall and Olkin (1979). Therefore, the normalized procedure used in the HDI does not matter in obtaining our dominance results.

4. Empirical application

To illustrate the conditions stated in the proposition discussed, we have considered the data used to build the HDI for 172 countries for the period 2000–2004.[5] No statistical inference is needed since we got almost the entire set of countries.

Our purpose is first to illustrate the analytical results. We also shed some light about the following hotly debated issue: Is globalization a process that creates winners and losers, and thus leads to greater inequality? Or in the contrary, are poor countries catching up not only in terms of income but also in terms of well-being? Since countries have very heterogeneous population sizes, weighing by population size may be important, even if it is not crucial for our empirical results interpreted qualitatively.

The results of our criteria show that one cannot say unambiguously that international welfare improved over the studied period, whether we use or not population weights. In particular, removing China and India from the database does not change this result. Even if the chosen one-dimensional welfare indicators have all much improved in these two countries, their contribution to the multi-dimensional dominance criteria is not sufficient to be able to state that the world situation is unambiguously better. Consequently and to simplify the exposition, we report the results only for the case where each country has the same weight. To be able to offer an accurate measure of inequality among the world's citizens, we would need to supplement the analysis with the knowledge of within-country inequality in the considered three dimensions. This is clearly out of the reach of this article.

The first element supporting our result of the absence of unambiguous welfare improvement is that the Generalized Lorenz curve for the health indicator (life expectancy at birth) at the final year lies systematically below that for the initial period as shown in Fig. 1. The expansion of the HIV/AIDS epidemic explains that, whereas only two countries had a life expectancy at birth below 40 in 2000 (Sierra Leone and Malawi), there were nine in 2004 (with in addition to the two previous ones: Rwanda, Zambia, Mozambique, Zimbabwe, Lesotho, Swaziland, and the Central African Republic). This finding confirms the results obtained by Decancq et al. (2009) over the period 1975–2000 using multidimensional inequality indices. After a decline over the first part of the period, health inequality among countries has surged since the nineties. However, when considering the population weighed case, the Generalized Lorenz curves intersect, and rising health country inequality can no longer be deduced from the data.

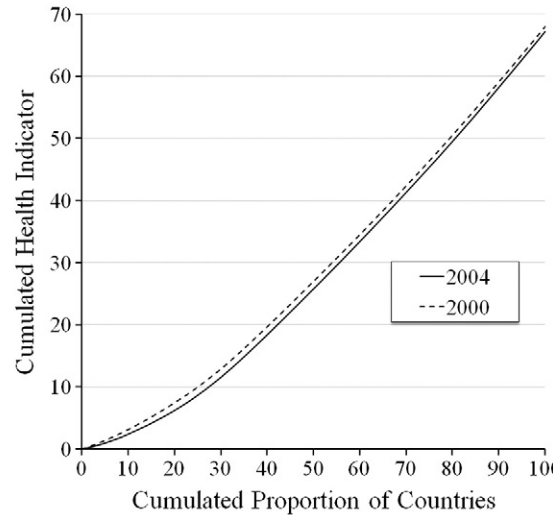

Fig. 1. Generalized Lorenz curves for health: condition (A).

On the other hand, the education distribution across countries has unambiguously improved over the period. Indeed, the Generalized Lorenz curve for 2004 lies unambiguously above that for 2000 as depicted in Fig. 2.

Even if one of the necessary conditions is not satisfied for international welfare improvement, we still pursue our investigation by looking at the other two conditions ((B) and (C)). For condition (B) applied to health, we rank the countries from the lowest health indicator. For condition (C) applied to education, we rank the countries from the lowest education indicator. We have already

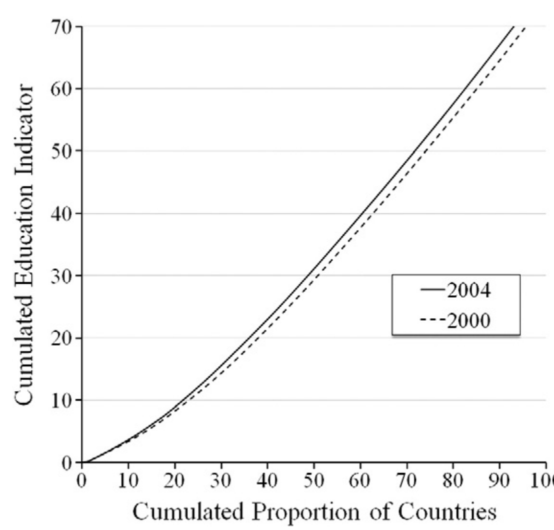

Fig. 2. Generalized Lorenz curves for education: condition (A).

[5] The data come from http://hdr.undp.org/statistics/data/.

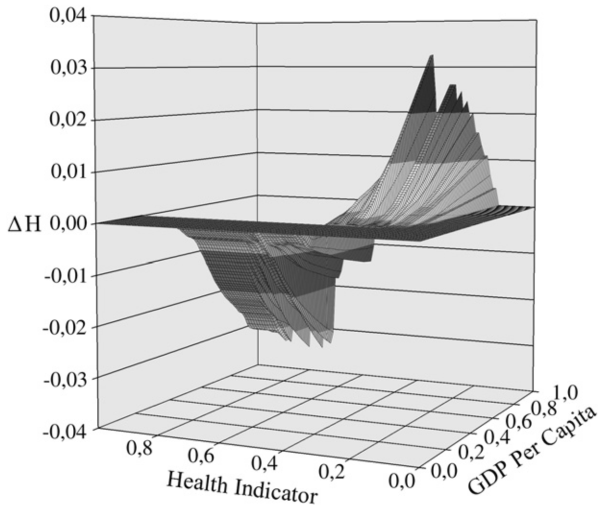

Fig. 3. Cross income–health condition (B); the front axis shows the health component, the receding axis shows the income component, the vertical axis shows the change in poverty gap.

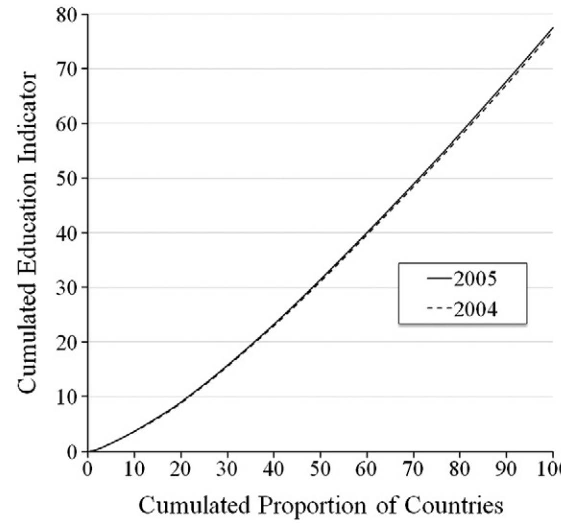

Fig. 5. Generalized Lorenz curves for education: condition (A) 2004/05.

seen that the health distribution has not unambiguously improved over the period.

Notwithstanding, we are also interested in knowing whether income has played some unambiguous role in compensating the possible worsening of the health distribution. If this had occurred, then the surface representing the differential between 2000 and 2004 in terms of conditions (B) and (C) would have been positive over the whole domain for x_2 and x_3. Fig. 3 reveals that this is not the case: the surface dives into negative values for low levels of the health indicator. This implies that some 'health-destitute' countries have seen their standard of living deteriorating as well.

Similarly, the cross income-education condition (C) is not satisfied. However, here only a small part of the surface remains out of the domain of positive values (see Fig. 4). Meanwhile, it can be noticed that standards of living only slightly improved for most 'education-destitute' countries.

We now consider the comparison between 2004 and 2005. The Generalized Lorenz curves for education and health both indicate that welfare considered purely in these dimensions has very slighty improved between these years (Figs. 5 and 6).

Then, we move to the examination of the stochastic surfaces. They clearly show that despite the favourable diagnosis provided by the Generalized Lorenz curves, it is not possible to conclude that the world's welfare has unambiguously improved between the two years (Figs. 7 and 8).

5. Conclusion

In this article, we have proposed a stochastic dominance approach to well-being across countries for monitoring international welfare. We consider a class of well-being indices based on the three attributes considered in the HDI: per capita GDP, life expectancy and educational attainment. We propose normative conditions that fit these variables: preference for more equal marginal distributions of income, health and education; ALEP substitutability among attributes and priority to poor countries in allocating international aid for improving health and education. These conditions define a class of well-being indices at the world level.

We derive three families of sufficient conditions in order to check dominance over this class. Of course, we do not claim that the exhibited conditions apply in any three-dimensional context. For instance, the additive separability assumption between health and education, which may seem reasonable for the case we studied, may prove to be too demanding in another setting. In that case, other

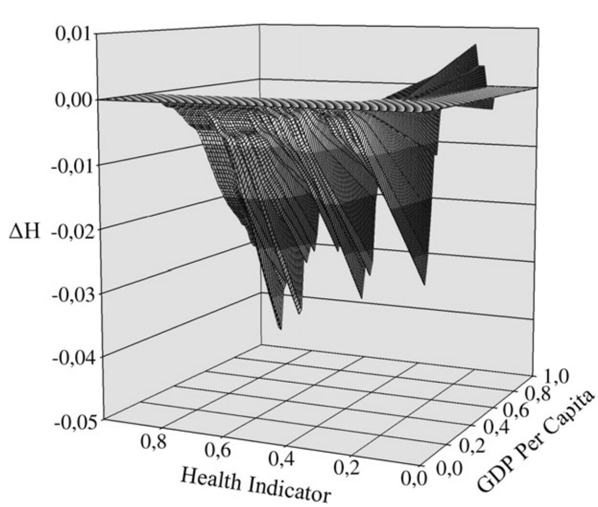

Fig. 4. Cross income–education condition (C); the front axis shows the education component, the receding axis shows the income component, the vertical axis shows the change in poverty gap.

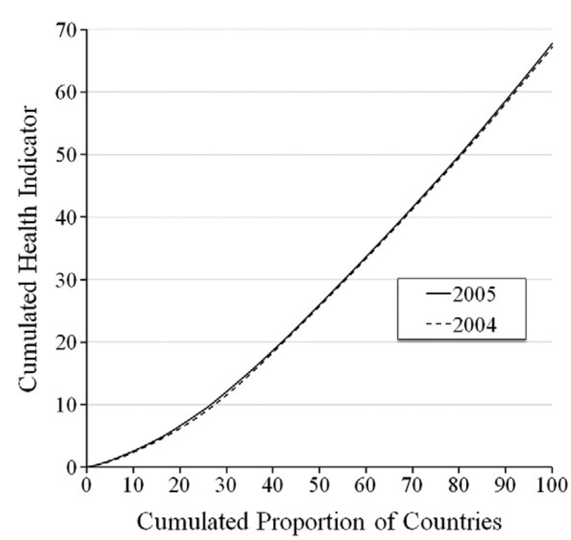

Fig. 6. Generalized Lorenz curves for health: condition (A) 2004/05.

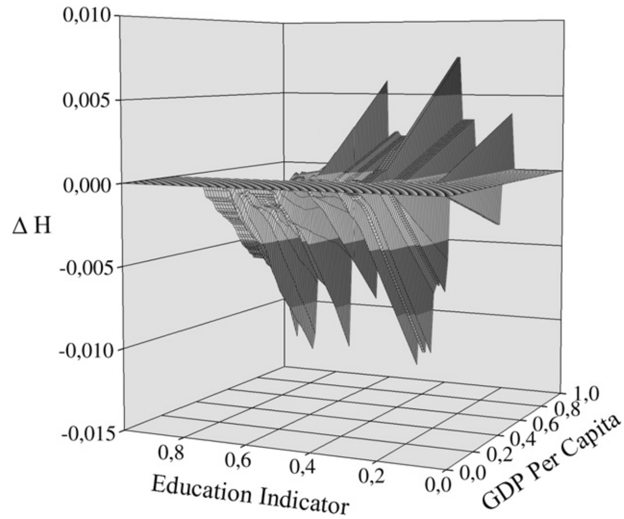

Fig. 7. Cross income–education condition (C) 2004/05; the front axis shows the education component, the receding axis shows the income component, the vertical axis shows the change in poverty gap.

appropriate conditions need to be designed to fit the context of interest. This is easy by following the method we proposed.

We applied our theoretical results to empirical appraisal of the change in the welfare distribution among countries between 2000 and 2004. Despite a globally improving situation, health conditions deteriorated so much in some poor countries that they make it impossible to conclude unambiguously that international welfare has improved over the period. Moreover, income only partly played its potential compensating role in these countries, and it was not enough to reverse the damaging effect of health deterioration on global welfare. In these conditions, more intensive health care and health prevention policies could be pursued by international organizations, accompanied by ambitious cash transfer policies to the ill.

Appendix A. Proofs

Proof of Proposition 1. The argument proceeds by integration by parts. The changes in ranks of integrations all along the proof with respect to the different variables are justified by Fubini theorem.

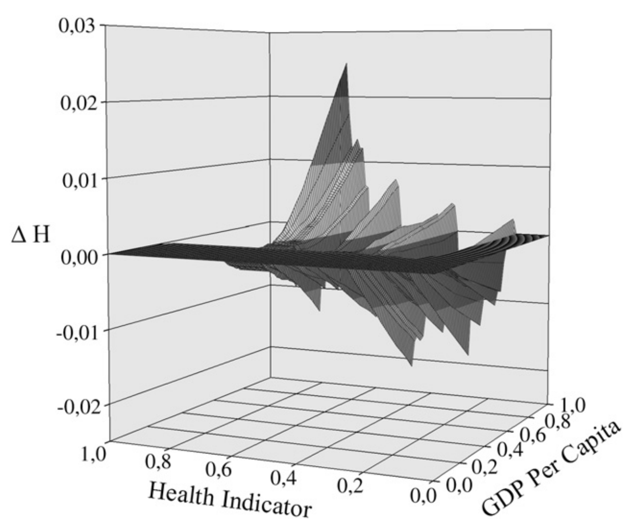

Fig. 8. Cross income–health condition (B) 2004/05; the front axis shows the health component, the receding axis shows the income component, the vertical axis shows the change in poverty gap.

Integrating the inner integral in Eq. (1) by parts with respect to x_1 gives.

$$\Delta W_I = \int_0^{a_3}\int_0^{a_2} I(a_1,x_2,x_3)\left[\int_0^{a_1}\Delta f(x_1,x_2,x_3)dx_1\right]dx_2dx_3 \tag{3}$$

$$-\int_0^{a_3}\int_0^{a_2}\int_0^{a_1} I_1(x_1,x_2,x_3)\left[\int_0^{x_1}\Delta f\left(s,x_2,x_3\right)ds\right]dx_1dx_2dx_3. \tag{4}$$

Let us call T_1 the first term (Eq. (3)) in the RHS term and T_2 the second term (Eq. (4)). It is convenient to treat separately these two terms. We start by integrating T_1 by parts with respect to x_3. We get

$$T_1 = \int_0^{a_2} I(a_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{a_1}\Delta f(x_1,x_2,x_3)dx_1dx_3\right]dx_2 - \int_0^{a_3}\int_0^{a_2} I_3(a_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{a_1}\Delta f(x_1,x_2,x_3)dx_1dt\right]dx_2dx_3.$$

We now integrate by parts the RHS of the expression discussed earlier with respect to x_2. We obtain

$$T_1 = I(a_1,a_2,a_3)\Delta F(a_1,a_2,a_3) - \int_0^{a_2} I_2(a_1,x_2,a_3)\Delta F(a_1,x_2,a_3)dx_2 - \int_0^{a_3} I_3(a_1,a_2,x_3)\Delta F(a_1,a_2,x_3)dx_3 + \int_0^{a_3}\int_0^{a_2} I_{32}(a_1,x_2,x_3)\Delta F(a_1,x_2,x_3)dx_2dx_3.$$

The first term in the RHS term is equal to zero because $F(a_1, a_2, a_3) = F^*(a_1, a_2, a_3) = 1$. Since $I_{32} = 0$, the last term vanishes. Finally, we integrate the second term of the RHS term of the above expression with respect to x_2 and the third term with respect to x_3. This yields

$$T_1 = -I_2(a_1,a_2,a_3)\Delta H_2(a_2;a_1,a_3) + \int_0^{a_2} I_{22}(a_1,x_2,a_3)\Delta H_2(x_2;a_1,a_3)dx_2 - I_3(a_1,a_2,a_3)\Delta H_3(a_3;a_1,a_2) + \int_0^{a_3} I_{33}(a_1,a_2,x_3)\Delta H_3(x_3;a_1,a_2)dx_3.$$

Let us now evaluate T_2. We start by integrating T_2 by parts with respect to x_3. We get

$$T_2 = -\int_0^{a_2}\int_0^{a_1} I_1(x_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{x_1}\Delta f(s,x_2,x_3)dsdx_3\right]dx_1dx_2 + \int_0^{a_2}\int_0^{a_3}\int_0^{a_1} I_{13}(x_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{x_1}\Delta f(s,x_2,t)dsdt\right]dx_1dx_3dx_2.$$

Integrating T_2 once more with respect to x_1 gives

$$T_2 = -\int_0^{a_2} I_1(a_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{a_1}\int_0^{x_1}\Delta f(s,x_2,x_3)dsdx_1dx_3\right]dx_2 + \int_0^{a_2}\int_0^{a_1} I_{11}(x_1,x_2,a_3)\left[\int_0^{a_3}\int_0^{x_1}\int_0^{s_1}\Delta f(s,x_2,x_3)dsds_1dx_3\right]dx_1dx_2$$
$$+ \int_0^{a_2}\int_0^{a_3} I_{13}(a_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{a_1}\int_0^{x_1}\Delta f(s,x_2,t)dsdx_1dt\right]dx_3dx_2 - \int_0^{a_2}\int_0^{a_3}\int_0^{a_1} I_{113}(x_1,x_2,x_3)\left[\int_0^{x_3}\int_0^{x_1}\int_0^{s_1}\Delta f(s,x_2,t)dsds_1dt\right]dx_1dx_3dx_2.$$

Finally, integrating T_2 with respect to x_2 yields

$$\begin{aligned}T_2 = & -I_1(a_1,a_2,a_3)\Delta H_1(a_1;a_2,a_3)\\ & + \int_0^{a_2} I_{12}(a_1,x_2,a_3)\Delta H_1(a_1;x_2,a_3)dx_2\\ & + \int_0^{a_1} I_{11}(x_1,a_2,a_3)\Delta H_1(x_1;a_2,a_3)dx_1\\ & - \int_0^{a_2}\int_0^{a_1} I_{112}(x_1,x_2,a_3)\Delta H_1(x_1;x_2,a_3)dx_1dx_2\\ & + \int_0^{a_3} I_{13}(a_1,a_2,x_3)\Delta H_1(a_1;a_2,x_3)dx_3\\ & - \int_0^{a_2}\int_0^{a_3} I_{123}(a_1,x_2,x_3)\Delta H_1(a_1;x_2,x_3)dx_2dx_3\\ & - \int_0^{a_3}\int_0^{a_1} I_{113}(x_1,a_2,x_3)\Delta H_1(x_1;a_2,x_3)dx_1dx_3\\ & + \int_0^{a_3}\int_0^{a_2}\int_0^{a_1} I_{1123}(x_1,x_2,x_3)\Delta H_1(x_1;x_2,x_3)dx_1dx_2dx_3.\end{aligned}$$

Therefore, the expression for the change in welfare becomes

$$\begin{aligned}\Delta W_I = & -I_1(a_1,a_2,a_3)\Delta H_1(a_1) - I_2(a_1,a_2,a_3)\Delta H_2(a_2)\\ & -I_3(a_1,a_2,a_3)\Delta H_3(a_3) + \int_0^{a_1} I_{11}(x_1,a_2,a_3)\Delta H_1(x_1)dx_1\\ & + \int_0^{a_2} I_{22}(a_1,x_2,a_3)\Delta H_2(x_2)dx_2 + \int_0^{a_3} I_{33}(a_1,a_2,x_3)\Delta H_3(x_3)dx_3\\ & + \int_0^{a_2} I_{12}(a_1,x_2,a_3)\Delta H_1(a_1;x_2)dx_2 + \int_0^{a_3} I_{13}(a_1,a_2,x_3)\Delta H_1(a_1;x_3)dx_3\\ & - \int_0^{a_3}\int_0^{a_1} I_{113}(x_1,a_2,x_3)\Delta H_1(x_1;x_3)dx_1dx_3\\ & - \int_0^{a_2}\int_0^{a_1} I_{112}(x_1,x_2,a_3)\Delta H_1(x_1;x_2)dx_1dx_2\\ & - \int_0^{a_2}\int_0^{a_3} I_{123}\left(a_1,x_2,x_3\right)\Delta H_1(a_1;x_2,x_3)dx_2dx_3\\ & + \int_0^{a_3}\int_0^{a_2}\int_0^{a_1} I_{1123}(x_1,x_2,x_3)\Delta H_1(x_1;x_2,x_3)dx_1dx_2dx_3.\end{aligned}$$

Since the well-being function satisfies $I_{23}=0$, and consequently $I_{123}=0$ and $I_{1123}=0$, the conclusion follows. □

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